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The Welfare Impacts of Competitive Telecommunications Supply: A Household-Level Analysis

THE TELECOMMUNICATIONS ACT of 1996 defines a regulatory framework for increasing the intensity of competition in all aspects of telecommunications supply. The act requires all regional Bell operating companies (RBOCs) to provide interconnection, unbundled access to their basic network elements, and resale of any retail services that they offer to potential competitive local exchange carriers. Once an RBOC meets a checklist of standards for providing access and interconnection services, the act allows it to ask the Federal Communications Commission (FCC) for permission to provide interLATA (Local Access and Transport Area) long-distance service.¹ The entry of competitive local ex-

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1. The 1982 Modification of Final Judgment (MFJ) that resulted in the breakup of AT&T into the seven regional Bell operating companies (RBOCs) and AT&T long-distance assigned all interstate and some intrastate long-distance services to the postdivestiture AT&T. The RBOCs were assigned local services, the remainder of intrastate toll services, and a very small amount of interstate toll services. The MFJ defined the geographic service areas of the RBOCs and AT&T in terms of local access and transport areas (LATAs). The boundaries of these geographic services areas were generally drawn to follow standard metropolitan statistical areas and not to cross state boundaries. Only AT&T Long-Distance and other long-distance carriers were allowed to provide long-distance service between LATAs across and within states (known as interLATA service). With a few exceptions the RBOCs were able to provide long-distance service within a

change carriers and, eventually, of RBOCs into the interLATA long-distance market should exert significant pressure on the cross-subsidies that currently exist in the pricing of telecommunications services.

One cross-subsidy, currently under intense scrutiny, flows from the charges long-distance service providers pay to the local exchange carriers at the originating and terminating points of a long-distance call. The rationale for this pricing of inter-exchange carrier access to the local telephone network is to maintain a low price for local residential service in order to achieve the goals of universal service.

During the years leading up to passage of the 1996 act, competition in the interLATA long-distance market and the growing number of competitive providers of long-distance access, particularly for large business customers, led to increasing RBOC revenue losses. Consequently, during this period, many RBOCs requested and some were granted increased prices for basic residential service as a way to recover these lost revenues.²

The increasing amount of competition in all telecommunications service markets envisioned by the 1996 law combined with the increasing penetration of competitive providers of access to long-distance service will put further pressure on all providers to price their telecommunications services to reflect the full cost of provision. The incumbent local exchange companies are aware of this logic and in their recent filings with the FCC, many have proposed increasing the price of local service by at least \$10 a month during the next several years to raise the price up to the cost of provision. Proponents of increasing the price of local service argue that such price increases will enable reductions in the prices of long-distance access and other services to reflect their costs so that the typical household's telephone bill may not rise by this

LATA (known as intraLATA long-distance service). The MFJ left the option with state regulatory commissions to determine whether to allow the RBOCs and the interLATA long-distance carriers to compete to provide intraLATA long-distance service within state boundaries. Weinhaus and Oettinger (1988) provide a nontechnical introduction to these and other issues associated with telecommunications regulation in the United States.

2. For example, effective January 1, 1995, the California Public Utilities Commission raised Pacific Bell's price of local residential service by approximately 35 percent (California Public Utilities Commission, 1994). Recently, US West made an unsuccessful request to the Arizona Corporation Commission to approve a revenue-neutral rate rebalancing plan that would shift some \$20 million in revenue away from intraLATA toll and onto local exchange service.

full \$10 and may in fact fall if the prices of these other services decline enough.

When considering whether to increase the price of local residential telephone service, two questions should be addressed. The first concerns the magnitude of the welfare loss that a household would experience if it had to pay more for local residential service. The second is whether increasing the price of local residential service together with reducing the price of long-distance access can achieve the household-level welfare benefits envisioned by its proponents. The purpose of this paper is to answer both of these questions at the level of the individual household.

The analysis focuses on the household because the regulatory debate over increasing the price of local residential service often revolves around the likely impacts of proposed price changes on specific demographic groups, such as low-income or elderly households. Even though the average household-level welfare loss from a proposed price change may be small, concern about the potentially large welfare losses to these groups often foils attempts to win regulatory approval for price increases. This concern also arises when considering proposals to balance increases in the price of local service with reductions in long-distance access prices. Critics argue that such policies increase the price of a good these households, particularly the elderly (for health and safety reasons), consider a necessity—local telephone service—and decrease the price of a good that they consider a luxury—long-distance service. By this logic, these households experience only the welfare loss of higher local phone service prices and little of the welfare gain of the decreased price of long-distance access, because they normally consume little, if any, long-distance service.

To quantify the change in a household's welfare attributable to an increase in the price of local service alone or an increase in the price of local service coupled with a decrease in the price of long-distance access requires an estimate of that household's indirect utility function. To obtain the required indirect utility function for all possible types of households, I specify and estimate a complete system of household-level demand functions that are derived from the assumption of static utility maximization.

The analysis of household-level demand is complicated by the fact that a significant fraction of households choose to consume none of one

or two of the goods in my demand system. These zero consumption levels are much more prevalent among low-income and elderly households, precisely those households most at issue in the regulatory debate over increasing the price of local residential service. The decision to consume or forgo consumption of any local phone service is equivalent to deciding whether to connect to the telecommunications network. Whether a household consumes any long-distance service at current prices is crucial to determining whether the household will receive any benefit from a decline in the price of long-distance service that accompanies an increase in the price of local service. Therefore, properly modeling both of these binary decisions—whether to consume zero or positive amounts of local service and long-distance service—is necessary to recover valid estimates of the welfare effects of price changes for those services, particularly for low-income households.

I also investigate a common assumption about the structure of household preferences for both local and long-distance service that can have a significant impact on the welfare calculations. Separability of local and long-distance telephone service from all other goods in the household's utility function implies that the household substitutes between local service and such goods as food or clothing in the same manner as it substitutes between long-distance service and either of these two goods. For the reasons given above, particularly for low-income or elderly households, this assumption is not likely to be true. Therefore, imposing it on the utility function of these households will lead to misleading welfare assessments for the price-change scenarios I consider. My econometric modeling framework and household-level database provide an opportunity to test this often maintained assumption, and I find substantial evidence against its validity.

My primary data source is the *Survey of Consumer Expenditures* (CES), put out by the Bureau of Labor Statistics (BLS). For this survey the BLS collects information on household consumption (broken down by many classes of goods), income, and various demographic characteristics, on a quarterly basis. From the first quarter of 1988 to the first quarter of 1991, this survey also collected data on household-level telephone consumption broken down by local and long-distance service, so my sample is confined to this time period. In addition to local and long-distance phone service, food, clothing, and other nondurable expenditures are included in the five-good demand system. To account

for across-household heterogeneity in preferences, I also include demographic variables interacted with prices in the indirect utility functions. I then estimate two models for the household-level indirect utility function, that account for zeros in the household's purchasing decisions in different ways.

For both models three classes of price-change scenarios are considered: those with a price increase in local residential service only, (2) those with a price increase in local service offset by an equal percentage decrease in the price of long-distance service, and (3) those with a price decrease for long-distance service that is twice the percentage increase in the price of local service. The second and third set of scenarios are designed to cover the range of price changes in local and long-distance services necessary to eliminate the current cross-subsidy in the pricing of local residential phone service. According to the estimated U.S. population aggregate household demand elasticities from my preferred model estimates, approximately equal and opposite percentage changes in the prices of local and long-distance service are necessary to keep aggregate U.S. household revenues to all local exchange carriers unchanged. Because there are reasonable demand elasticity estimates that imply the decrease in long-distance prices would have to be twice as large as the increase in local service prices to keep revenues unchanged, I also analyze the welfare implications of this type of price-change scenario.

For the price-change scenarios that exactly balance a price increase against a price decrease—for instance, a 20 percent increase in the price of local service coupled with a 20 percent reduction in the price of long-distance service—parameter estimates from my boundary model of household choice imply an average household-level welfare loss. To emphasize the importance of properly accounting for zeros in household-level consumption patterns, the interior solution model—my other model of household choice—yields the opposite conclusion: an average household-level welfare gain from this pair of price changes. For the two scenarios where the price decrease for long-distance service is twice that of the price increase for local service, both models of household choice estimate an average welfare gain. The boundary solution model, which accounts for zeros in a more realistic manner, predicts substantially smaller household-level gains, however. Because it treats zero consumption in a more comprehensive manner than the

interior solution model does, the boundary solution model results in very different estimates of the price and expenditure elasticities of clothing and long-distance service (two goods with a significant number of zeros) and overwhelmingly rejects separability of the underlying household-level utility function.

For local price increases unaccompanied by decreases in the price of long-distance service, both models estimate relatively minor welfare losses. For example, a 40 percent increase in the price of local service only results in a sample average household-level quarterly compensating variation of \$17.86 in January 1988 dollars for the boundary model for the household's indirect utility function. This figure is approximately 0.6 percent of the sample mean of household total nondurable expenditure, which is \$3,017.52 in January 1988 dollars. The sample 5th percentile to 95th percentile range of compensating variations for this price change for the boundary model is \$13.06 to \$23.12, so even for these extremes of the sample, the welfare losses associated with these price changes seem relatively minor. Even for a price increase of this magnitude (40 percent), for both model estimates I do not find any households in the sample consuming a positive amount of local service before the price change having a predicted consumption of local telephone service less than or equal to zero, which can be thought of as disconnecting from the local exchange network.

Because the empirical analysis recovers an estimate of each household's indirect utility function, I can compute an estimate of the utility-constant or true cost-of-living increase associated with each price-change scenario for each household in the sample. For the 40 percent increase in the price of local service, the mean household-level true cost-of-living increase is 1.08 percent for the boundary model estimates, with a 5th to 95th percentile range of 0.65 percent to 1.87 percent. The largest household-level true cost-of-living increase from the 40 percent increase in the sample is 3.0 percent. These results indicate that a price increase for local service would lead to little loss of household-level welfare for all but a small fraction of U.S. households, and little, if any, reduction in the fraction of households connected to the local telephone network.

Because the CES associates with each sample household a weight estimating the number of U.S. households with the same demographic composition as that household, I can compute the weighted sum of

household-level compensating variations, which gives the total aggregate net benefit or loss to society from each price-change scenario. Dividing this magnitude by the sum of the weights—an estimate of the total number of households—yields the U.S. population average per-household compensating variation. A final exercise I undertake with these weights is to estimate the median compensating variation for the U.S. population of households. I find that these estimated U.S. population calculations agree with the analogous sample calculations in both sign and magnitude.

Finally, I estimate how the burden associated with each of these price change scenarios is shared across the various types of households in the sample. Measuring this burden by the percent true cost-of-living increase, I find that the burden is much more than proportionately borne by the lowest total expenditure (income) segment of the population of U.S. households. In addition, this burden is also borne to a greater extent by older-headed households, those with a working head and working spouse, those in urban areas, and those with more children ages two through fifteen.

Increasing Competition and Cross-Subsidies in Telecommunications Markets

In the years just before passage of the Telecommunication Act of 1996, an increasing number of competitive providers of access to long-distance services emerged; most of these providers served large business customers, traditionally the major source of revenues for local exchange carriers. The geographic concentration of local exchange business revenues in large urban centers makes relatively small-scale entry by competitive access providers extremely profitable. For example, US West, the RBOC serving Washington state, estimates that 30 percent of its business calling revenues come from customers in 0.1 percent of the land area of the state.

Significant revenue losses in the long-distance access market have forced the RBOCs to seek, and many have already received, regulatory approval for increased prices for local residential phone service. As a result, the portion of total revenues that all U.S. local exchange carriers

received from providing local phone service increased from 41.7 percent in 1988 to 45.6 percent in 1994.³

During this same time period, a growing number of states decided to allow competition in intraLATA toll service, historically a significant revenue source for local exchange carriers. At the present time, all states allow some form of intraLATA long-distance competition from facilities-based long-distance carriers. Many states have implemented or are in the process of implementing plans for intraLATA long-distance dialing parity, where customers can presubscribe to the intraLATA toll carrier of their choice and all “1 +” intraLATA long-distance calls will automatically be handled by that carrier. These changes in the intraLATA market have resulted in substantial toll revenue losses to local exchange carriers: in 1988, 16.8 percent of all revenues for the RBOCs and independent local exchange carriers came from toll service; by 1994, that figure had fallen to 13.4 percent.⁴

Competition among toll carriers has also cut into AT&T's interstate market share measured in minutes, which dropped from more than 85 percent in 1985 to less than 60 percent in 1994.⁵ For my purposes, however, a more important aspect of this long-distance market is the extent to which reductions in the price long-distance carriers pay to access the local exchange network are passed through to consumers in the form of lower prices for long-distance service. The question has been debated extensively. After an exhaustive survey of the literature and some analysis of their own, Crandall and Waverman conclude that virtually all of the reductions are passed through—a reduction of one cent in access prices translates into an eventual one-cent reduction in long-distance prices—for daytime interstate rates for the longer mileage distances in the U.S. interstate long-distance market.⁶ Consequently, it seems reasonable to assume that a regulatory policy that increases the price of local service and reduces the price of access for long-distance providers will eventually result in the desired reduction in the price of long-distance service.

Between 40 and 45 percent of the total cost of an interLATA long-distance call is paid to the local exchange carriers at the originating and

3. United States Telephone Association (1989, 1995).

4. Ibid.

5. Crandall and Waverman (1995, pp. 30, 137).

6. Crandall and Waverman (1995).

terminating points of the call. Although these access fees are generally thought to be higher than their cost, there is some disagreement over just how much higher they are. According to Sievers, approximately half of these payments to local exchange carriers are in excess of the costs of local access.⁷ In a recent decision involving US West, the Washington Utilities and Transportation Commission (WUTC) concluded that “it is not a matter of dispute that access charges greatly exceed the incremental cost of access.”⁸ The WUTC decision also found that local residential service was not priced below its average incremental cost, and that the price provided a substantial contribution to shared and common costs. The question still remains, however, whether the revenues from local residential service are sufficient to cover all of the costs remaining after other revenue sources have been applied.

In their recent Universal Service filings with the FCC, all of the RBOCs except NYNEX (which serves New England and New York) favored substantial increases in the price of local residential service in line with what they argue is the cost of providing this service. Mary McDermott, vice president of regulatory policy for the United States Telephone Association, stated, “Right now the average local rate is about \$18 [a month]. Over the next four or five years it’s reasonable to think of a \$28 basic rate.”⁹ She goes on to state that because the prices of long-distance access and other services will fall to reflect their costs, the typical consumer’s overall telephone bill may not rise by this full \$10 and may in fact fall, if the prices of these other services decline enough.

The provisions of the Telecommunications Act of 1996 that allow interconnection and access to basic network elements by the competitive local exchange carriers will exert further pressure on the prices of all services to reflect their costs. The new law explicitly states that the rates charged to competitive local exchange carriers must be “based on the cost (determined without reference to a rate-of-return or other rate-based proceeding) of providing the interconnection or network ele-

7. Sievers (1994).

8. WUTC (1996, p. 110).

9. Mike Mills, “Phone Rates Face New Hike Proposals,” *San Jose Mercury News*, May 7, 1996, p. A1.

ment.”¹⁰ In addition, the eventual entry of the RBOCs into the interLATA long-distance market will make the rapid pass-through of access charge reductions even more likely.

Although substantial increases in the price of local residential service are not inevitable, it is difficult to believe, given the new competitive circumstances, that the RBOCs will not incur substantial revenue losses if the price of local service is not increased significantly. Consequently, it seems reasonable to believe that price increases along the lines of the price-change scenarios set out here are likely to be seriously debated in all of the coming state regulatory proceedings associated with implementing the Telecommunications Act of 1996 and in subsequent price-setting proceedings. The recent WUTC decision is a case in point: US West proposed a doubling of the rate for local residential service over four years.

Research Question within the Context of the Existing Literature

Many studies have assessed the impact of local service price increases on telephone subscribership rates. Other studies have examined the structure of telecommunications demand.¹¹ Two recent studies attempt to quantify the welfare gains associated with an increase in the price of local service coupled with a decrease in the price of long-distance service. Crandall and Waverman consider the overall gains to society, to telecommunications services producers, and to consumers from making these two price changes in a manner consistent with Ramsey pricing. Using aggregate demand elasticities and estimates of the marginal cost of supplying local and long-distance service, they compute the aggregate welfare under Ramsey pricing. After comparing this result to aggregate welfare at the current prices, they find significant gains to society from this rate rebalancing process.¹²

Gabel and Kennet consider this same question and come to a different conclusion. They argue that because of the impact of varying technical standards on the cost of service, the marginal cost of local exchange

10. Telecommunications Act of 1996, section 252(d)(1)(a)(i).

11. Taylor (1994) provides a book-length survey of these studies.

12. Crandall and Waverman (1995). The authors obtain their aggregate demand elasticities from the studies surveyed in Taylor (1994).

service has been overstated and that the elasticity of demand for long-distance service has been overstated in absolute value. They claim that the combination of these two circumstances leads to no clear welfare gains associated with reductions in the price of long-distance service accompanied by increases in the price of local service.¹³

These two studies focus on the aggregate demand for local and long-distance service. For the many studies of the impact of local and long-distance prices on subscribership, the analysis typically focuses on subscribership rates within a census block group or other geographic region. For long-distance consumption the analysis typically focuses on aggregate demand for a state or for the nation as a whole.¹⁴ Any estimate of the change in aggregate household welfare must therefore be derived from the aggregate Marshallian demand curve, which could yield aggregate welfare change estimates wildly different from the sum of the individual household-level compensating variations associated with the price-change scenario under consideration.

Consequently, this study focuses on characterizing the structure of household-level demand and on measuring household-level welfare in a manner consistent with economic theory. I chose the compensating variation relative to the actual prices faced by a household as the measure of the welfare change associated with a given price-change scenario. The utility-constant cost-of-living increase associated with the price-change scenario is then computed for each household in the sample. In addition, the CES sampling weights permit estimates of the U.S. population household-level mean and median compensating variation and true cost-of-living increase. Because the regulatory debate over the impact of the price changes I consider usually concentrates on their effect on low-income and elderly households or those from disadvantaged ethnic groups, it is especially important to measure welfare changes at the household level to understand whether state regulatory commissions would be willing to implement these local residential service price increases.

Because many of the households in these demographic groups of particular concern to state regulators consume little or no local and long-distance service, properly accounting for the presence of zero

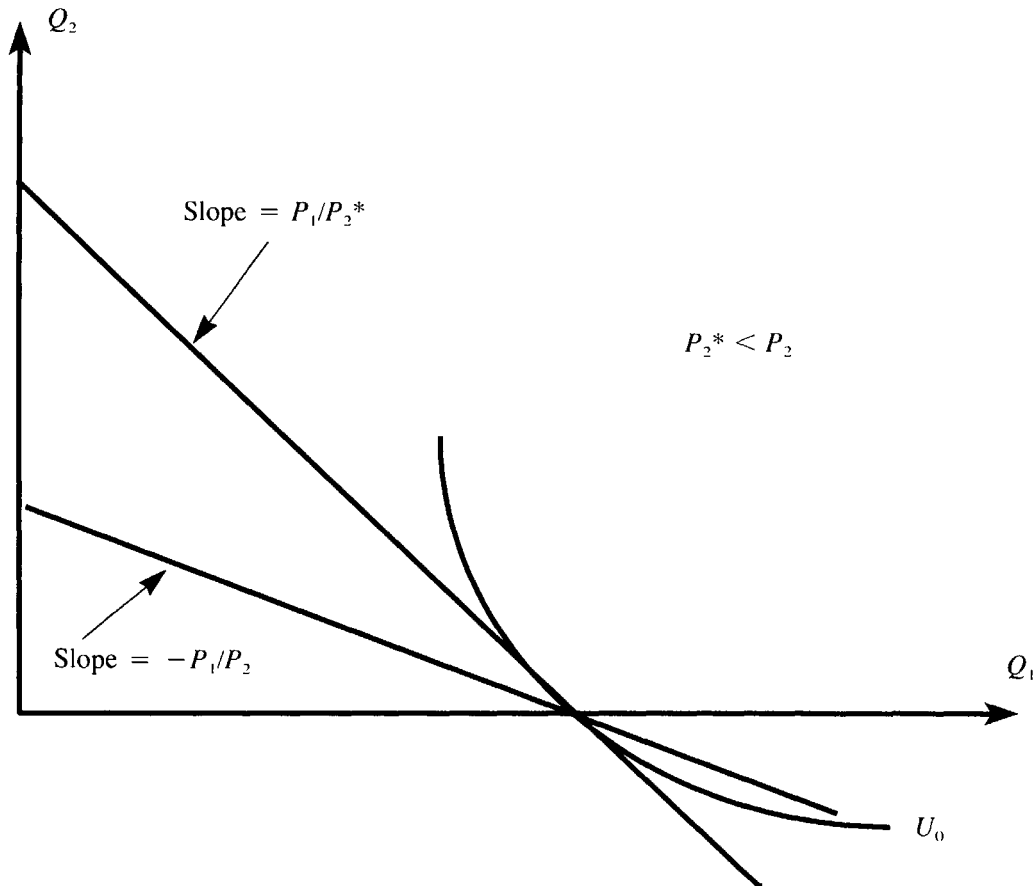
13. Gabel and Kennet (1993).

14. See Crandall and Waverman (1995) for specific examples of these studies.

consumption in the household's demand is crucial to correctly recovering the household's indirect utility function. I take two approaches to accounting for zeros. The tradeoff between the two is theoretical consistency of the demand system against the stringency of the statistical assumptions and ease of estimating the resulting demand system. The first approach treats zero consumption as an interior solution to the household's budget-constrained utility-maximization problem. In other words, a household's decision not to consume a good is treated the same as other nonzero consumption choices the household makes, which is why I call it the interior solution approach. The second approach acknowledges that zero consumption is attributable to the imposition of the Kuhn-Tucker conditions for nonnegativity of the utility-maximizing vector of expenditure shares. Here, the econometric model of household-level demand explicitly accounts for these Kuhn-Tucker conditions for observations with zero consumption of one or more goods. I call this the boundary solution approach. Although this approach is theoretically superior to the interior solution approach, the explicit accounting for the nonnegativity constraints on household demands requires explicit specification of distributional assumptions and considerably complicates the estimation of the resulting demand system.

Besides its consistency with the household's nonnegativity constrained utility maximization problem, I expect that the boundary solution model will more accurately capture the welfare effects I would like to measure for those households of particular concern to state and federal regulators. Consider figure 1, which illustrates the household's nonnegativity constrained choice of long-distance service and all other goods at the prevailing prices P_1 (the price of all other goods) and P_2 (the price of long-distance service). As the chart is drawn, the household is at a corner solution at utility level U_0 with zero consumption of long-distance service. Consequently, for price reductions up to the level of P_2^* , the household will still be at a corner solution and will optimally choose only to consume all other goods. Consequently, any price decrease for long-distance from P_2 to a price greater than or equal to P_2^* will result in no change in the household welfare because the utility-maximizing consumption choice will not change. In contrast, the interior solution model assumes that the observed zeros in the household's consumption choice are unconstrained utility-maximizing choices, so

Figure 1. Corner Solution in Actual Price (P_2) and Interior Solution in Virtual Price (P_2^*)



Source: See text for definitions and explanation.

any reduction in the price of long-distance service for this model will result in welfare improvements to the consumer.

Whether separability of local and long-distance service from all other goods is imposed on the household's utility function can have large effects on household-level welfare calculations. Both models show substantial evidence against the null hypothesis of separability of local and long-distance service from all other goods. This result implies that, at least for my dataset, to estimate consistently the parameters of the household-level demand for local and long-distance service, a two-stage budgeting approach that specifies the demand for local and long-distance service independent of the prices and expenditure on all other goods should not be used.

Distribution of Household-Level Telephone Expenditures

The dataset used here consists of quarterly observations on consumption expenditures for local telephone service, long-distance service, food, clothing, and other nondurable goods. The demand analysis focuses on total nondurable consumption to avoid the issues associated with the distinction between the price of the current period's service flow from a good and the purchase price of that good. This distinction arises whenever the good purchased provides services for a longer time than the period in which the purchase is observed (in this case a quarter). By definition, this type of good is a durable good, hence my focus on total nondurable consumption, which henceforth is referred to as total expenditure.

The CES defines local telephone service as all expenditures for local telephone service for that household. It includes the cost of local phone service for all phones in all dwellings the household owns and any installation charges associated with these phones that occur within the sample quarter. Long-distance telephone service consumption is defined as the total of all long-distance calling charges where the cost of a single call is broken out in detail on the phone bill. Food consumption is defined as all expenditures on food consumed both within and outside the household (restaurant meals, for example). Clothing consumption is the total of all clothing purchases made by the household during that quarter. Other nondurable consumption is defined as spending on commodities such as gasoline, household heating fuel, electricity, transportation services, and other nondurable consumption services. For this analysis, all nominal magnitudes have been deflated to January 1988 dollars using the total nondurable goods price index from the BLS *Consumer Price Index Detailed Report*.

To illustrate the features of the dataset that drive the demand system estimation results, the distributions of local, long-distance, and total telephone expenditures are decomposed across the sample of households by the quartiles of the total expenditure distribution. Total expenditure rather than the household income is used because for a large fraction of households, income in a given time period is a very poor predictor of the household's total consumption expenditures during that period. For the usual life-cycle, permanent-income considerations, total

expenditure in any period is likely to be more highly correlated with permanent income than with actual income for that period.¹⁵

Quarterly Telephone Expenditures

The quartiles of the total quarterly expenditures distribution for my sample of 11,467 households and the mean level of spending for each quartile, in January 1988 dollars, are as follows:

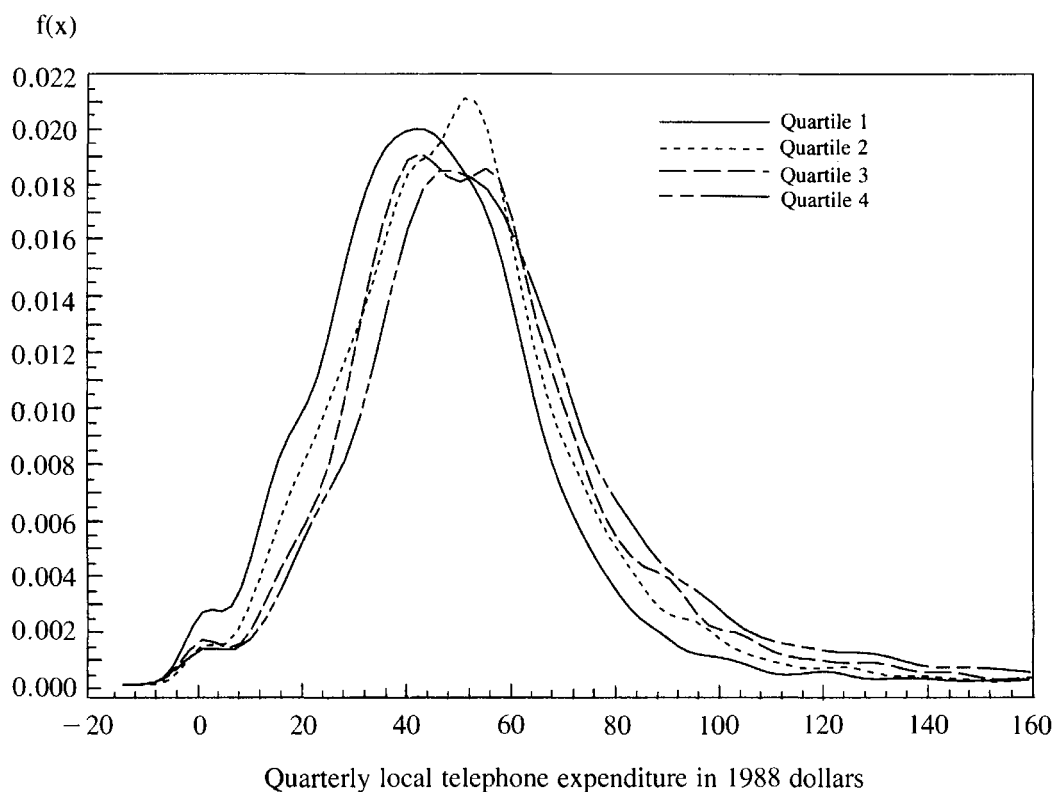
<i>Quartile</i>	<i>Total quarterly expenditures range</i>	<i>Mean</i>
1	Less than 1,688.00	1,192.43
2	1,688.00–2,594.36	2,126.83
3	2,594.37–3,787.27	3,127.46
4	More than 3,787.27	5,624.48

For each of these total expenditure quartile subsamples, I compute a kernel estimate of the density of expenditures on local, long-distance, and total telephone service.¹⁶ Figure 2 plots estimates of the density of total quarterly local telephone expenditure for all of the quartiles of the total expenditure distribution. The striking aspect of this figure is that densities of local telephone expenditures do not vary much across the quartiles of the total expenditure distribution. Even a comparison of households in the first quartile with those in the fourth quartile reveals small differences in the densities for local service expenditures. A very different story emerges in figure 3, which plots kernel estimates of the density of total quarterly expenditures on long-distance service. Each density is generally shifted to the right of the density for the expenditure quartile below it and is less positively skewed. This pattern indicates a relatively expenditure-elastic demand for long-distance service. For total quarterly telephone expenditures, given in figure 4, the density

15. This point is discussed by Lusardi (1993) for the CES and by Blundell, Pashardes, and Weber (1993) for the *Family Expenditure Survey*, the United Kingdom's analogue to the CES.

16. Silverman (1986) provides a comprehensive discussion of kernel density estimation. I use a Gaussian kernel and the automatic bandwidth selection procedure Silverman recommends for all of the density estimates presented in this paper. Note, however, that because of the local smoothing property of the kernel estimation process, these densities can take on positive values for negative values of telephone expenditure even though the actual data contain no negative values. In the limit, as the number of observations tends to infinity, this estimated positive probability mass on negative values would tend to zero.

Figure 2. Densities of Household-Level Local Telephone Expenditure in January 1988 Dollars by Quartiles of Total Nondurable Expenditure Distribution



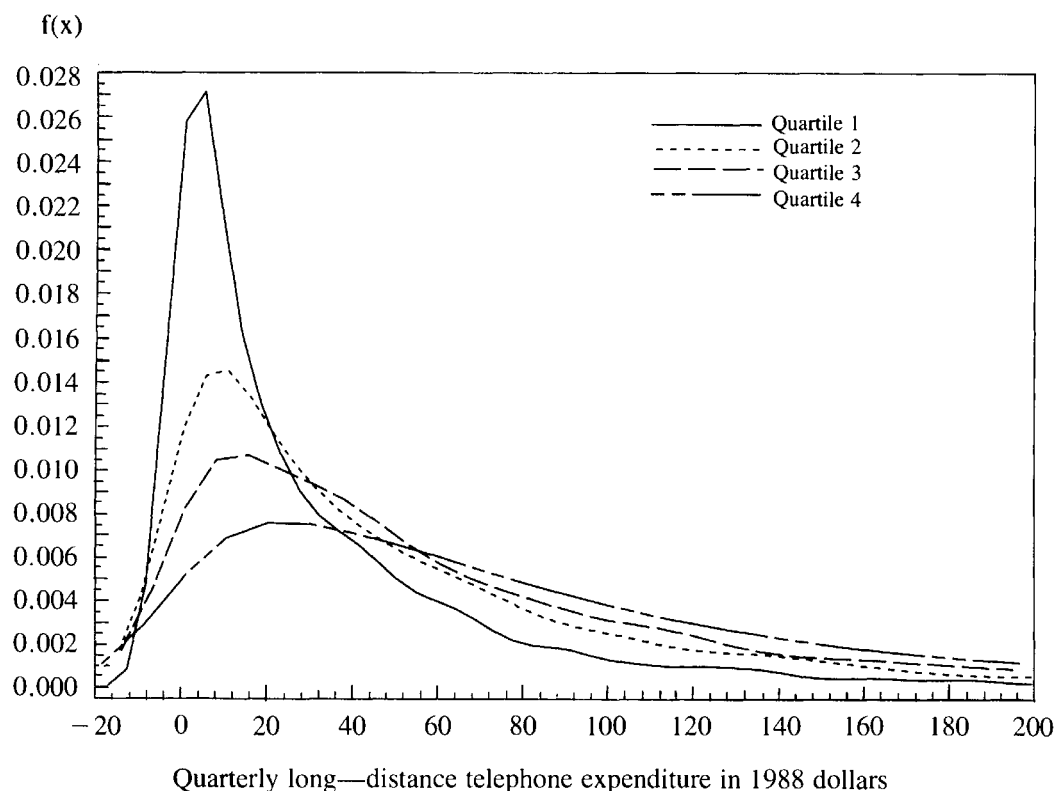
Source: Author's calculations.

shifts across expenditure quartiles in a pattern similar to the pattern for long-distance service, although the increase in the positive skewness associated with moving to a higher total expenditure quartile is much less pronounced. The sample correlation between local and long-distance telephone expenditures within the household is 0.17, which implies a surprisingly small degree of positive linear dependence between these two components of the total telephone bill and explains why the pattern of density shifts for total phone expenditures is less pronounced than for long-distance expenditures.

Price Data Used in the Analysis

A major problem faced by all demand system studies using household-level data is the lack of a cross-section dataset of commodity prices that can be linked to the sample of households; thus all price series used

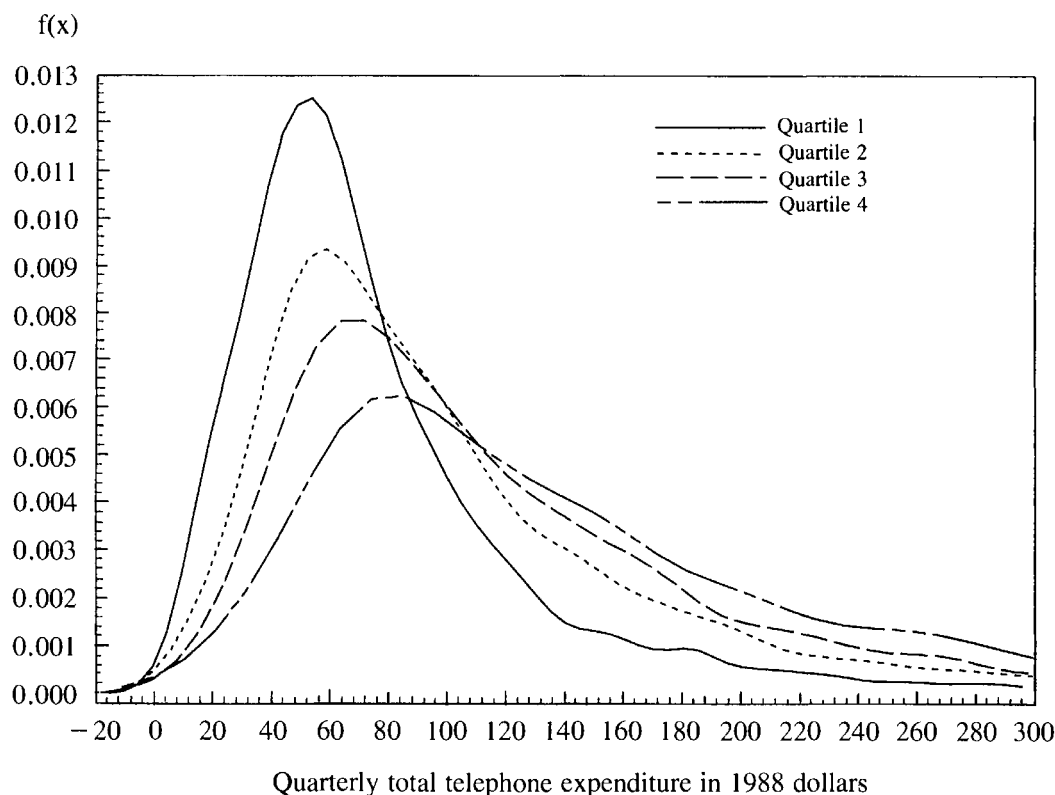
Figure 3. Densities of Household-Level Long-Distance Telephone Expenditure in January 1988 Dollars by Quartiles of Total Nondurable Expenditure Distribution



Source: Author's calculations.

in these analyses vary only over time. Even if commodity-specific price data were available at some degree of cross-sectional disaggregation, say, at the state level, the analysis would be complicated by confidentiality considerations that preclude the BLS from releasing any state identifiers for a substantial fraction of household-level observations (approximately a quarter of the households in this analysis, for example). Until very recently, the BLS did not release information on state identifiers for any households. The degree of geographic detail available for all household observations is the census region. Although the BLS does compute price indexes for food, clothing, and other nondurable goods on a monthly basis for each of the four census regions, it computes price indexes for local and long-distance service on a monthly basis only at the national level. It is plausible to assume that all households face the same price for long-distance service because this product is sold by firms serving the national market. The state-level regulatory

Figure 4. Densities of Household-Level Total Telephone Expenditure in January 1988 Dollars by Quartiles of Total Nondurable Expenditure Distribution



Source: Author's calculations.

price-setting process makes this assumption less tenable for local phone service, however. Nevertheless, the considerable amount of implicit and explicit cross-state communication among the regulatory bodies in setting local service prices argues in favor of assuming that local prices move together over time.

An additional problem with assigning local service prices to specific households is that the price for local service often depends on where in the local exchange carrier's network a household is located. Many states set the price of local service at the wire center level for each local exchange carrier, depending on the supposed cost characteristics of that wire center. Consequently, knowing the state or even the town in which a household lives may not be enough to assign the correct price for local service to many households in the sample.

Despite these arguments in favor of treating local service prices as if they moved together over time for every household in the sample, I

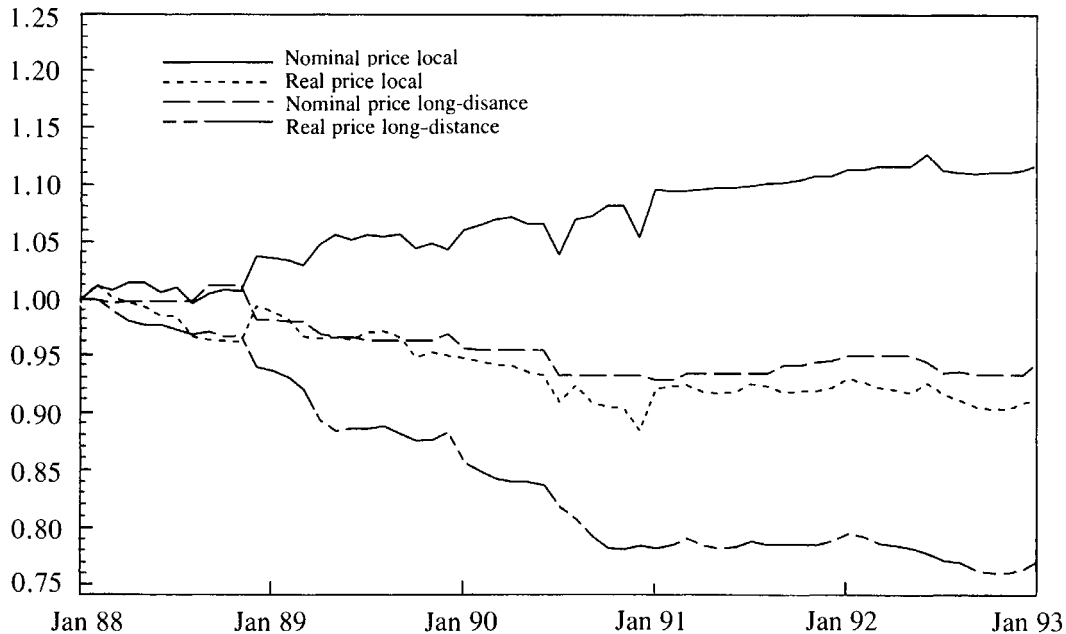
must acknowledge that for data availability reasons and confidentiality considerations, the analysis makes this unsatisfying assumption. To mitigate the effect of these data limitations, I introduce the possibility of unobservable household-level prices for both local and long-distance service into the boundary solution model of demand. In this way I account for the concern that different households can face different prices for local and long-distance service and that these differences may be one reason why two very similar households—in terms of observable characteristics—consume very different amounts of local and long-distance service.

The maximum amount of meaningful price variation is introduced into the analysis by using all available cross-sectional and time series variation in the BLS price indexes. For food, clothing, and other non-durable goods, I use the BLS price indexes for the census region in which that household resides. For confidentiality reasons the BLS does not report a census region of residence for rural households, so I use the national price indexes for these three goods in this case. The initial level of relative prices across the four census regions is unknown, because, by construction, all four regional indexes are normalized to equal one in the same base period; four census region dummies are therefore included in each of the expenditure share equations. (The excluded region is the rural region.) It is straightforward to show that by including these regional dummies in each demand equation, I account for the unknown sample differences in relative prices across regions in the base periods.

Although only a national price index is available for local and long-distance service, the rolling panel nature of the CES data collection process enables me to introduce some across-household variation for the prices of these two goods and for the prices of food, clothing, and other nondurable goods within a given quarter. Data for the CES is collected on a monthly basis for each household for the consumption amounts in the previous quarter. During any month, therefore, a different set of households is being retrospectively interviewed for their consumption patterns in the previous quarter. Households usually remain in the survey for three quarters and then exit. Because the questionnaire is retrospective and because data for the BLS price-index series are collected on a monthly basis, I use the price index for local and long-distance phone service for the most recent month of the pre-

Figure 5. Monthly Price Indexes for Nominal and Real Local and Long-Distance Telephone Service

Price Index



Source: Bureau of Labor Statistics; see text for explanation.

vious quarter. For the three price indexes available at the census region level, I use the value for the most recent month for the census region in which that household resides. I deflate all five of the nominal monthly commodity price indexes by the total nondurable consumption price index described above to obtain real price indexes keyed to a January 1988 base period, so that the expenditure and price figures are comparable.

Figure 5 plots the monthly nominal and real BLS price indexes for local and interstate long-distance phone service from January 1988 to January 1993.¹⁷ (Recall that the real and nominal series for both local and long-distance phone service are normalized to one in January 1988.) The figure shows that although the nominal price of local phone service

17. The time series pattern of the BLS intrastate long-distance price index closely tracks the BLS interstate long-distance price index over the sample period. The interstate long-distance price index is used in this analysis because the vast majority of long-distance calls are interstate calls. Experimenting with alternate composite indexes of these two long-distance price indexes did not significantly change any of my estimation results.

increased over this time period, the real price remained almost constant throughout this four-year period. The long-distance price index showed modest nominal declines and substantial real declines during the sample period. These price trends have continued for local service. Since discount residential long-distance calling plans became widely available in 1992, long-distance carriers have questioned the accuracy of the long-distance price index. Fortunately, the sample period ends in the first quarter of 1991, so this price index accuracy issue should not affect my econometric modeling results.

Econometric Modeling Framework for Interior Solution Model

The econometric modeling framework must be sufficiently flexible to encompass several empirical and theoretical considerations. The first empirical consideration is the long history of work indicating nonhomothetic preferences. The most well-known result along these lines is Engel's Law, which states that the proportion of a household's spending devoted to food decreases as its total spending increases. Recent research provides evidence against traditional Engel curve representations with budget shares as linear in the log of total expenditure, as first specified by Working and Leser.¹⁸ Given the strong empirical evidence against homotheticity, I must select an underlying household-level utility function that is nonhomothetic and allows for budget shares that are nonlinear functions of the log of total expenditure.

From the theoretical perspective, there are several requirements for my underlying utility function. The first is the ability to impose the restrictions implied by utility-maximizing behavior on the demand functions estimated in a data-independent fashion. The second requirement is second-order flexibility of the underlying utility function, which essentially means that for any point in the data space, the functional form can exactly reproduce any theoretically possible value of the function, its gradient, and matrix of second-partial derivatives through appropriate choice of the parameters of the functional form. The final theoretical requirement is the ability to impose the restrictions implied

18. See Working (1943) and Leser (1963). Deaton and Muellbauer (1980) survey the evidence against homothetic preferences. Bierens and Pott-Buter (1990) and Hausman, Newey, and Powell (1995) are representative of the most recent line of research.

by separability of local and long-distance phone service from all other commodities in a data-independent manner using restrictions on the parameters of the demand system. In this way, the null hypothesis of separability can be tested using conventional parametric hypothesis testing techniques.

The translog is one functional form that satisfies these theoretical and empirical considerations. I utilize duality theory to recover the parameters of the indirect utility function from the Marshallian demand functions because I have observations on price indexes associated with the five goods consumed. (Quantity indexes would be required to recover estimates of the parameters of the direct utility function for the cases in which the underlying utility functions are not self-dual in the sense discussed by Houthakker.¹⁹)

I now describe the translog indirect utility function and discuss how to impose the restrictions implied by optimizing behavior on the demand functions estimated. These restrictions are, first, homogeneity of degree zero of the demand functions in prices and total expenditure; second, symmetry of the Slutsky matrix (the matrix of compensated own- and cross-price effects); and, third, quasi-convexity of the indirect utility function in the prices, which is equivalent to negative semi-definiteness of the Slutsky matrix.

In the following notation, let p_i denote the price of good i , x_i the quantity of good i consumed, and M the total expenditure. In this notation

$$M = \sum_{i=1}^N p_i x_i \text{ and } w_i = \frac{p_i x_i}{M},$$

where w_i is the share of total expenditure spent on the i th good and N is the total number of goods consumed. The translog indirect utility function for this notation is

$$(1) \quad \ln[V(P, M)] = \alpha_0 + \sum_{i=1}^N \alpha_i \ln\left(\frac{p_i}{M}\right) + \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \beta_{ij} \ln\left(\frac{p_i}{M}\right) \ln\left(\frac{p_j}{M}\right)$$

where $P = (p_1, p_2, \dots, p_N)$ is the vector of prices for the N goods. Ap-

19. Houthakker (1965). Jorgenson and Lau (1975) provide a detailed discussion of the data requirements for estimating direct and indirect utility functions for general preference structures.

plying the logarithmic version of Roy's Identity to this indirect utility function yields share equations

$$(2) \quad w_i(P, M) = \frac{\alpha_i + \sum_{j=1}^N \beta_{ij} \ln\left(\frac{p_j}{M}\right)}{\sum_{k=1}^N \alpha_k + \sum_{k=1}^N \sum_{j=1}^N \beta_{kj} \ln\left(\frac{p_j}{M}\right)} \quad (i = 1, \dots, L).$$

Roy's Identity implies that if $V(P, M)$ is a proper indirect utility function, then the right-hand side of equation 2 is the value of $w_i(P, M)$ that maximizes utility subject to the household's budget constraint. Unfortunately, Roy's Identity does not imply the constraint that this maximizing value of $w_i(P, M)$ is nonnegative. This possibility is accounted for explicitly in the boundary solution model, but for the interior solution model, I must assume that the application of Roy's Identity to the indirect utility function at the prevailing prices and total expenditure level always yields nonnegative shares.

Because share equation 2 is homogeneous of degree zero in the parameters α_i and β_{ij} , a single normalization restriction must be imposed to identify the remaining parameters. The usual restriction is to impose

$$\sum_{i=1}^N \alpha_i = -1.$$

By inspection, the share equation is homogeneous of degree zero in the vector of prices P and total expenditure M , so that homogeneity imposes no restrictions on the parameters of the model. The Slutsky matrix is

$$(3) \quad S = \Pi^{-1} \left[\frac{1}{D(P, M)} (I - \mathbf{w} \mathbf{w}')' \Delta_{pp} (I - \mathbf{w} \mathbf{w}') + \mathbf{w} \mathbf{w}' - W \right] \Pi^{-1},$$

where Π is the $(N \times N)$ diagonal matrix with (p_i/M) as the i th diagonal element, $\mathbf{w} = (w_1, w_2, \dots, w_N)$, Δ_{pp} is an $(N \times N)$ matrix with β_{ij} as the (i, j) th element, W is the $(N \times N)$ diagonal matrix with w_i as the i th diagonal element and $\mathbf{1}$ is an N -dimensional vector of ones.²⁰ The func-

20. For more details, see the discussion of the Slutsky matrix in Jorgenson, Lau, and Stoker (1982).

tion $D(P, M)$ is the denominator of the fraction on the right-hand side of equation 2. This expression implies that symmetry of S is equivalent to the symmetry of Δ_{pp} , which holds if $\beta_{ij} = \beta_{ji}$ for all i and j . Equation 3 also shows that the quasi-convexity of the indirect utility function in prices (which implies that S is negative semi-definite) is a data-dependent restriction. The S explicitly depends on the observed prices, shares, and total expenditure. Consequently, my strategy is to estimate the model without imposing this restriction. Given the parameter estimates obtained, I check to see whether this constraint holds for each of the points in that dataset before performing the welfare calculations for that observation, because it makes little economic sense to perform these welfare calculations for observations failing the conditions for integrability.

Because across-household differences in consumption patterns are used to identify the parameters of the demand systems, I want to distinguish between consumption differences attributable to differences in prices and total expenditure and those attributable to differences in household characteristics. For this reason household demographic characteristics are included in the translog indirect utility function. For these differences to be econometrically identified, they must enter interacted with functions of prices and total expenditure. Defining A_k as the k th demographic characteristic and A as the K -dimensional vector of these characteristics, the translog indirect utility function with demographic characteristics becomes

$$(4) \quad \ln[V(P, M, A)] = \alpha_0 + \sum_{i=1}^N \alpha_i \ln\left(\frac{p_i}{M}\right) + \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \beta_{ij} \ln\left(\frac{p_i}{M}\right) \ln\left(\frac{p_j}{M}\right) + \sum_{i=1}^N \sum_{k=1}^K \eta_{ik} \ln\left(\frac{p_i}{M}\right) A_k.$$

Applying the logarithmic version of Roy's Identity, the translog share equations become

$$(5) \quad w_i(P, M, A) = \frac{\alpha_i + \sum_{j=1}^N \beta_{ij} \ln\left(\frac{p_j}{M}\right) + \sum_{k=1}^K \eta_{ik} A_k}{\sum_{k=1}^N \alpha_k + \sum_{k=1}^N \sum_{j=1}^N \beta_{kj} \ln\left(\frac{p_j}{M}\right) + \sum_{i=1}^N \sum_{k=1}^K \eta_{ik} A_k}.$$

The assumption of utility-maximizing behavior does not impose any restrictions on the η_{ik} . To compute the Slutsky matrix for this demand system, the variable $D(P, M)$ in equation 3 now becomes $D(P, M, A)$ and is given by the denominator of equation 5.

To estimate the interior solution demand system, I must specify a stochastic structure that accounts for differences between the observed expenditure shares and those predicted by the share equations in equation 5. To allow for these differences, I append to the share equations additive mean zero errors ϵ_i , which can be correlated with the errors from the other share equations for a given household, yet are independently distributed across households. This implies that the observed expenditure shares, w_i ($i = 1, \dots, N$), satisfy the equation $w_i = w_i(P, M, A) + \epsilon_i$, where $w_i(P, M, A)$ is defined in equation 5. If $\epsilon = (\epsilon_1, \epsilon_2, \dots, \epsilon_N)$ is the vector of share disturbances for a given household, I assume that $E(\epsilon) = 0$ and $E(\epsilon\epsilon') = \Omega(P, M, A)$, where $\Omega(P, M, A)$ is some matrix that depends on the prices, total expenditure, and household characteristics associated with that observation. I interpret ϵ as the unobservable (to the econometrician) portion of the household's indirect utility function, so the general form for the household-level indirect utility function is $V(P, M, A, \epsilon)$. Brown and Walker show that under this interpretation for the stochastic structure of a demand system and associated indirect utility function, the vector of additive share equation disturbances, ϵ , should be heteroskedastic conditional on prices and total expenditure.²¹ For this interior solution version of the demand system, I must assume that the application of the logarithmic version of Roy's Identity to $\ln[V(P, M, A, \epsilon)]$ yields observed optimizing shares that are nonnegative. To be more precise, the interior solution model assumes that a household's observed vector of demands, x^* , is the solution to

$$\max_x U(x, A, \epsilon) \text{ subject to } \sum_{i=1}^N p_i x_i = M,$$

where $U(x, A, \epsilon)$ is the household's direct utility function. The interior solution model imposes the household's budget constraint only on the set of feasible demands.

Choosing a specific functional form for the way in which ϵ enters

21. Brown and Walker (1989).

the indirect utility function would imply a specific functional form for the dependence of Ω on P and M . This dependence could be exploited to yield more efficient estimates of the parameters of the model. If the functional form for the matrix $\Omega(P, M, A)$ is incorrect, however, this approach would adversely effect the consistency of the estimates of the parameters of the indirect utility function. Consequently, my strategy is to assume that a portion of the household's indirect utility function is unobservable so that the utility function takes the form $V(P, M, A, \epsilon)$ and to acknowledge the results of Brown and Walker in the estimation process. I do not, however, explicitly model how ϵ enters the indirect utility function. Instead, I require only that it enter in a way that yields additive disturbances to the share equations that satisfy the moment restrictions for ϵ given above.

To investigate the empirical importance of these considerations, I test for the existence of heteroskedasticity (which depends on P and M) in disturbances to the share equations. If there is evidence of the dependence of Ω on these variables, rather than selecting a parametric model for this dependence and reestimating the model, my strategy is to construct standard error estimates that are consistent in the presence of this form of heteroskedasticity. As a result, although the parameter estimates would be less efficient, all of the inferences would be based on asymptotically valid standard error estimates. Because the parameters of the interior solution version of the demand system can be consistently estimated by imposing only the assumed orthogonality restrictions between ϵ and the log-prices, log-total expenditure, and the demographic characteristics, this modeling approach seems to be the best research strategy to balance the competing goals of parametric flexibility and precise estimation of the parameters of the model given my relatively large sample.

The final issue in making the stochastic structure consistent with utility maximization is that summability of the observed budget shares implies that the sum of the ϵ_i over all goods is identically zero. In terms of the earlier notation, this restriction is $\iota' \epsilon = 0$, which implies $\iota' \Omega(P, M, A) \iota = 0$. This restriction turns $\Omega(P, M, A)$ into a matrix of rank $N - 1$. To estimate this model, I simply drop one of the share equations and estimate parameters of the demand system using the remaining $N - 1$ system of share equations. So long as I use a quasi-likelihood

function approach to estimate the model, the parameter estimates will be invariant to the share equation that is dropped.

I use the multivariate normal density to construct the quasi-likelihood function.²² This quasi-maximum likelihood approach is a general estimation strategy that is consistent with utility-maximizing behavior, yet one that does not require a specific distributional assumption for ϵ or a parametric form for the dependence of Ω on P , M , and A to obtain consistent estimates of the parameters of the demand system and to make asymptotically valid inferences about the parameters of the demand system.

Defining the multivariate normal quasi-likelihood function requires the following notation. Define y_j to be the $(N-1)$ -dimensional vector of expenditure shares for the j th household and $f_j(P^j, M^j, A^j, \theta)$ to be the $(N-1)$ dimensional vector of fitted expenditure shares, which are functions of prices, expenditure, and household characteristics for the j th household. Let J denote the total number of households in the sample. In this notation θ denotes the vector of parameters of the demand system to be estimated. To compute the quasi-maximum likelihood estimate of θ , I maximize

$$L(\theta, \Omega) = -J(N-1)\ln(2\pi) - \frac{J}{2}\ln\det(\Omega) \\ - \sum_{j=1}^J \frac{1}{2}[y_j - f_j(P^j, M^j, A^j, \theta)]' \Omega^{-1} [y_j - f_j(P^j, M^j, A^j, \theta)]$$

with respect to θ and the parameters of the matrix Ω .

To test the null hypothesis of homoskedastic disturbances to the share equations, I perform the kurtosis-consistent version of the Bruesch and Pagan Lagrange Multiplier test for homoskedasticity against the alternative that the variance of the disturbances to each share equation depends on prices and total expenditure, as the results of Brown and Walker imply.²³ This test is implemented by taking the residuals from the quasi-maximum likelihood estimation of each of the share equations

22. Gouriéroux, Monfort, and Trognon (1984) prove the consistency of these quasi- (or pseudo-) maximum likelihood estimators for the parameters of my demand system, and White (1982) provides consistent standard error estimates under the moment conditions I specify for ϵ .

23. Bruesch and Pagan (1979) and Brown and Walker (1989).

and regressing these residuals squared on a constant, the log prices, log total expenditure, and all of the unique cross-products of these log prices and log total expenditure. Taking J times the R^2 from this regression yields the Lagrange Multiplier statistic, which is asymptotically distributed as χ^2 with degrees of freedom equal to $\frac{1}{2}[(N+1)^2 - (N+1)] + 2(N+1)$, where N is the number of goods in the model. For my five-good model, this number is 27.

Econometric Modeling Framework for Boundary Solution Model

The major difference between the interior solution model and the boundary solution model is that the latter explicitly accounts for the fact that at the prevailing prices, a household may have a notional demand for some good—that obtained from maximizing utility subject only to the household's total expenditure constraint—that is negative. For those households, the requirement that demands must be nonnegative implies that the household's observed vector of demands, x^* , is the solution to

$$(6) \quad \max_{x \geq 0} U(x, A, \epsilon) \text{ subject to } \sum_{i=1}^N p_i x_i = M.$$

The boundary solution model explicitly imposes nonnegativity constraints, in addition to the household's budget constraint, on the set of feasible demands.

This model accounts for the possibility given in figure 1. Zero consumption occurs because the actual prices of nonconsumed goods are greater than the virtual prices for these goods—those prices that yield unrestricted demands for nonconsumed goods exactly equal to zero. A rigorous definition of virtual prices follows. Writing the objective function for optimization problem 6 yields

$$(7) \quad Z = U(x, A, \epsilon) + \lambda \left(M - \sum_{i=1}^N p_i x_i \right) + \sum_{i=1}^N \psi_i x_i,$$

where λ is the Lagrange Multiplier associated with a household's budget constraint and the ψ_i ($i = 1, \dots, N$) are Kuhn-Tucker multipliers associ-

ated with the nonnegativity constraints on the demands x_i ($i = 1, \dots, N$). Assuming that the household does not consume the first L goods ($x_i^* = 0$ for $i = 1, \dots, L$), the first-order conditions for equation 7 are

$$\frac{\partial U(x, A, \epsilon)}{\partial x_i} - \lambda p_i + \psi_i = 0, \quad \psi_i \geq 0, \quad i = 1, \dots, L,$$

$$\frac{\partial U(x, A, \epsilon)}{\partial x_j} - \lambda p_j = 0, \quad j = L + 1, \dots, M,$$

$$\text{and } \sum_{j=L+1}^N p_j x_j = M, \quad \lambda > 0.$$

Neary and Roberts²⁴ show that if $U(x, A, \epsilon)$ is continuous and strictly monotonic, then the virtual prices for the first L goods are

$$\begin{aligned} (8) \quad p_i^v(p) &= p_i - \frac{\psi_i}{\lambda} = \frac{\partial U(x, A, \epsilon)}{\partial x_i} \bigg/ \lambda \\ &= p_N \frac{\partial U(x, A, \epsilon)}{\partial x_i} \bigg/ \frac{\partial U(x, A, \epsilon)}{\partial x_N}. \quad (i = 1, \dots, L) \end{aligned}$$

Under these conditions on $U(x, A, \epsilon)$, the demand for good i is zero if and only if the virtual price for the good, $p_i^v(p)$, is less than or equal to the actual price of the good, p_i , because $\lambda > 0$ (by strict monotonicity of the utility function) and $\psi_i \geq 0$ if and only if the constraint $x_i \geq 0$ holds as the equality $x_i = 0$.

Note that a household's utility function depends on ϵ , the unobserved portion of the household's utility function, so determining whether a virtual price for a good is less than or equal to its actual price—whether the household consumes some quantity of this good—requires explicit specification of the dependence of the utility function on the vector ϵ . Values for these disturbances that imply virtual prices less than actual prices yield zero realized demands. Determining the likelihood function value for households with zero consumption of one or more goods requires integrating over the set of values of ϵ , giving rise to virtual prices less than or equal to actual prices for those goods not consumed by that household. Therefore both the dependence of the indirect utility

24. Neary and Roberts (1980).

function on the disturbances and a specific distributional assumption for these disturbances are required to compute the likelihood function necessary to estimate the parameters of the household's indirect utility function for the boundary solution approach. In contrast to the interior solution model, the boundary solution model has a firm foundation in economic theory, with the cost being more stringent assumptions on the stochastic structure of the model.

The development of the likelihood function follows the general approach given in Pitt and Lee, who present a theoretical framework for estimating demand systems with binding nonnegativity constraints.²⁵ I use the dual approach and specify an indirect utility function where the dependence of $\ln[V(P, M, A, v)]$ on v , the N -dimensional vector of disturbances, takes the following form:

$$\begin{aligned}
 \ln[V(P, M, A, v)] = & \alpha_0 + \sum_{i=1}^N \alpha_i \ln\left(\frac{p_i}{M}\right) \\
 (9) \quad & + \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \beta_{ij} \ln\left(\frac{p_i}{M}\right) \ln\left(\frac{p_j}{M}\right) \\
 & + \sum_{i=1}^N \sum_{k=1}^K \eta_{ik} \ln\left(\frac{p_i}{M}\right) A_k + \sum_{i=1}^N v_i \ln\left(\frac{p_i}{M}\right).
 \end{aligned}$$

Applying the logarithmic version of Roy's Identity to this indirect utility function yields a household's potentially negative virtual demands:

$$(10) \quad w_i^*(P, M, A, v_i) = \frac{\alpha_i + \sum_{j=1}^N \beta_{ij} \ln\left(\frac{p_j}{M}\right) + \sum_{k=1}^K \eta_{ik} A_k + v_i}{\sum_{k=1}^N \alpha_k + \sum_{k=1}^N \sum_{j=1}^N \beta_{kj} \ln\left(\frac{p_j}{M}\right) + \sum_{i=1}^N \sum_{k=1}^K \eta_{ik} A_k}.$$

When these share equations are related back to those given in equation 5, it can be seen that the additive disturbance for the interior solution model, ϵ_i , equals $v_i/D(P, M, A)$, where $D(P, M, A)$ is the denominator for the right-hand side of equation 10. This is consistent with the results of Brown and Walker who find that an additive disturbance to the share

25. Pitt and Lee (1986).

equation must be heteroskedastic conditional on prices, total expenditure, or both to be consistent with the hypothesis of utility maximization.²⁶ I impose the same normalizations on the α_i (their sum over i equals -1) and on the ν_i (their sum over i equals 0) as in the interior solution model. The restriction on the ν_i implies that the covariance matrix of this N -dimensional vector has rank $N-1$. Let Σ denote the covariance matrix of the $N-1$ dimensional vector $\nu = (\nu_1, \nu_2, \dots, \nu_{N-1})$.

Because the translog indirect utility function satisfies the Neary and Roberts regularity conditions given above, the decision rule determining zero consumption of a good by a household is that the good's virtual price is less than or equal to its actual price. In this case, the demands for the remaining goods are derived from the budget-constrained utility maximization problem subject to the constraint of zero demands for those goods whose virtual price is less than the actual price. Equivalently, as equation 8 illustrates, the remaining demands can be determined from substituting the virtual prices for the goods not consumed and the actual prices of goods consumed into the demand functions for the goods that are consumed. If all virtual prices are greater than their respective actual prices, then all observed demands are positive.

The use of virtual prices is nothing more than a simplified mathematical technique for deriving the nonnegativity-constrained utility-maximizing level of demand for a specific value of ν , given a household's stochastic indirect utility function. Because I can observe which of the goods the household does not consume, I know that the virtual price for each of these goods is less than the actual price. This inequality constraint on the virtual demands, however, gives only a set of inequalities on the errors to the share equations with zero expenditures. The remaining equations of the demand system also contain these same share equation errors because the virtual prices, which depend on these errors, enter each of these equations. Consequently, to compute the likelihood function value for households with zero observed demands for some goods, I must integrate with respect to the unobserved share equation errors, ν_i , over the set of values that generate virtual prices less than or equal to actual prices for those goods with zero observed demands.

Appendix 1 describes the computation of the likelihood function for

26. Brown and Walker (1989).

three classes of observations—no zeros, one zero, and two zeros. No observations in the sample have more than two zeros. The case of no zeros is very similar to the quasi-likelihood function given for the interior solution model, with the exception of a Jacobian term because the boundary solution model requires an explicit specification of the dependence of additive share equations on P , M , and A . Computing the likelihood function for a single zero requires a univariate integration with respect to v_i , the disturbance to the expenditure share equation with zero demand. Two zeros require a bivariate integration with respect to v_i and v_j , the disturbances to the two expenditure share equations with zero demands. The likelihood function for the complete sample is the product of likelihoods associated with all of the households in the sample.

To complete the specification of the likelihood function, I assume that there is a joint distribution of the prices of local and long-distance service for the sample of households. The observed price indexes for local and long-distance service are assumed to be the means of these distributions for each household. Let p^l denote the observed price of local service and p^d the observed price of long-distance service to a household. Specifically, I assume that the actual prices faced by the household satisfy the relations $p(act)^l = p^l\theta$ and $p(act)^d = p^d\gamma$, where $E(\theta) = 1$ and $E(\gamma) = 1$ and the random vector (θ, γ) is independent and identically distributed across households. I further assume that (θ, γ) possesses a discrete distribution $\{\delta_i, \theta_i, \gamma_i\}$ ($i = 1, \dots, L$). Because there is good reason to believe that the distribution of prices differs across households in urban versus rural areas, I use two different discrete joint distributions for household-level local and long-distance price heterogeneity depending on whether the household is located in a rural or an urban area.²⁷ To determine the likelihood function for a single observation under this assumption, for no, one, or two zeros, I compute the summation

27. Telephone companies serving sparsely populated rural areas primarily serve residential customers, while companies in urban metropolitan areas obtain a significant fraction of their revenues from business customers and network services not demanded in rural areas. These differences in pricing incentives and product offerings should result in different distributions of across-household price heterogeneity for local service and long-distance service for these two geographic areas.

$$(11) \quad L_z(w|\Gamma, \Sigma, \Delta) = \sum_{k=1}^H \delta_k^r L_z(p^r \theta_k^r, p^d \gamma_k^r, p^f, p^c, p^o, M, A, \Gamma, \Sigma),$$

where the price vector faced by the household is $P = (p^r \theta^r, p^d \gamma^r, p^f, p^c, p^o)$, $L_z(w|P, M, A)$ is one of the three likelihood function values given in appendix 1 for $z = 0, 1$, or 2 , and Δ is a $2 \times 3H$ -dimensional vector of the δ_i^r , θ_i^r , and γ_i^r . The r accounts for my assumption of different distributions of price heterogeneity in urban versus rural areas; these differences mean that the likelihood function also differs across these two geographic areas. I assume the value $H=2$ in the estimation for both the urban and rural price heterogeneity distributions. The log-likelihood function for all observations is the logarithm of the sum of these terms, where L_z is replaced by the value relevant for the number of zeros in each observation ($z=0,1,2$). I then maximize the log-likelihood function with respect to the parameters Γ (of the indirect utility function), Σ (covariance matrix of v), and Δ (the parameters of the joint distributions of (θ^r, γ^r) , $r = \text{urban and rural}$) subject to the constraints

$$\sum_{k=1}^H \delta_k^r = 1, \sum_{k=1}^H \delta_k^r \theta_k^r = 1, \text{ and } \sum_{k=1}^H \delta_k^r \gamma_k^r = 1,$$

which correspond to the constraints that probabilities sum to one, $E(\theta^r) = 1$ and $E(\gamma^r) = 1$, for both rural and urban households. Consequently, the estimation procedure also accounts for the potential that different households face different prices and that the across-household distribution of these prices differs between urban and rural households.

The estimation is substantially more computationally intensive than for the interior solution model because computing the likelihood function contribution for households with zero consumption in one or two goods requires the computation of either a univariate or bivariate integral, and these observations make up more than 10 percent of the sample. Table 1 gives the breakdown of zeros for the sample. The diagonal elements give the number of single-zero observations for each good, and the off-diagonal elements, the number of two-zero observations for each combination of row and column label. The vast majority of single zeros are associated with long-distance expenditures. Clothing

Table 1. Joint Empirical Distribution of Zero Expenditures

<i>Good</i>	<i>Local</i>	<i>Long-distance</i>	<i>Food</i>	<i>Clothing</i>	<i>Other</i>
Local service	146				
Long-distance	4	999			
Food	0	0	0		
Clothing	8	209	0	712	
Other	0	0	0	0	0
Total	158	1,208	0	712	0

Source: Author's calculations

Note: 11,467 total observations; 9,389 all-positive observations.

is a close second, and local service a distant, but still significant, third. There are a few two-zero observations, with the most common pair being long-distance service and clothing purchases. Because of this fact, the boundary solution and interior solution models recover very different preference structures for these two goods, and these differences have important implications for the subsequent welfare calculations for the multiple price-change scenarios. The significant number of households consuming zero of at least one good points out the potential importance of properly accounting for these boundary solutions in the empirical work. I now turn to a discussion of my empirical results for both models.

Estimation Results

I now describe the specific household characteristics entering into the models, the estimation procedures and their output, and the results of the separability tests for both models.

In choosing household characteristics to include in the vector $A = (A_1, A_2, \dots, A_K)$, I attempt to control for across-household differences in the preferences for goods that do not depend on the prices faced by the household or its total expenditure. The age and age-squared of the head of household are included, because I anticipate cohort differences in the demand for telephone service. Dummies for whether the household contains a working head and a working spouse are included, because there is strong evidence (which I confirm) that employment status and

hours of work affect consumption patterns.²⁸ For this same reason I also include variables measuring the number of hours worked annually by the head and by the spouse.

To account for differences in the geographic location of households to the extent possible given confidentiality constraints and to account for the unobservable base-period across-region relative prices for food, clothing, and other nondurable goods discussed earlier, I include a dummy for each of the four census regions, with the excluded group those households living in rural areas. Because I believe that a household's demographic composition influences its preferences, I include variables measuring the total number of people in the household, the number of people sixty-five years old and over, and the number of males and the number of females between two and fifteen years old. Dummy variables indicate race and educational status; specifically, I include dummy variables for whether the head of household is white, a high school graduate, and a college graduate. Finally, I include indicator variables for whether the household head is a female, single, and works in a professional occupation as defined by the CES. Although other household characteristics could have been included, preliminary model estimations indicate that these variables are sufficient, relative to models with more household characteristics included, to explain much of the across-household differences in consumption not attributable to differences in prices and total expenditure.

Table 2 presents the quasi-maximum likelihood estimates of the parameters of the interior solution demand system in terms of the notation for the translog indirect utility function given in equations 4 and 5. Table 2 also contains the maximum likelihood parameter estimates for the boundary solution model in terms of the notation given in equations 9 and 10. Both models are estimated with the summability, homogeneity, and symmetry restrictions imposed so that the resulting demand systems can be used to perform welfare calculations. Formal statistical tests of these restrictions yield little evidence against their validity for either model. Despite the price data used, both models yield fairly precisely estimated own-price effect coefficients and some precisely estimated cross-price effects. The very small standard errors relative to

28. See, for example, Browning and Meghir (1991).

Table 2. Coefficient Estimates for Interior and Boundary Solution Models

Coefficient	Estimates (standard errors)	
	Interior solution model	Boundary solution model
$\beta_{Mi} = \sum_{j=1}^N \beta_{ij}$, assuming $\beta_{ij} = \beta_{ji}$		
α_L	0.0186 (0.00154)	-0.0210 (0.000521)
α_D	-0.0349 (0.00391)	-0.0164 (0.00107)
α_F	-0.277 (0.0188)	-0.318 (0.00583)
α_C	-0.152 (0.0120)	-0.117 (0.00452)
β_{LL}	-0.00207 (0.0112)	-0.00686 (0.000979)
β_{LD}	-0.00102 (0.00471)	0.00505 (0.000431)
β_{LF}	-0.00646 (0.0101)	-0.00549 (0.00188)
β_{LC}	-0.00527 (0.00538)	-0.0000714 (0.000935)
β_{DD}	0.0190 (0.00592)	0.00612 (0.000527)
β_{DF}	-0.0348 (0.0141)	-0.00120 (0.00150)
β_{DC}	-0.0126 (0.00849)	-0.00405 (0.000638)
β_{FF}	0.122 (0.0643)	-0.0275 (0.0233)
β_{FC}	-0.0671 (0.0313)	-0.00878 (0.0108)
β_{CC}	0.107 (0.0277)	0.0365 (0.00900)
β_{ML}	-0.00992 (0.00154)	-0.00394 (0.000120)
β_{MD}	0.000407 (0.000762)	-0.0000436 (0.000201)
β_{MF}	0.0342 (0.00862)	0.0231 (0.00105)
β_{MC}	0.0494 (0.00714)	0.0284 (0.000797)
β_{MO}	0.159 (0.0231)	0.0870 (0.00119)

(continued)

Table 2. Continued

Coefficient	Estimates (standard errors)	
	Interior solution model	Boundary solution model
<i>Local share equation</i>		
$\eta_{Lk}, k = 1, 2, \dots, K$		
Age of head $\times 10^{-2}$	-0.000811 (0.0102)	0.0277 (0.00191)
(Age of head) ² $\times 10^{-4}$	-0.000661 (0.0104)	-0.0290 (0.00207)
Number in household	-0.00112 (0.000302)	0.000283 (0.0000699)
Members ≥ 65 years old	-0.000348 (0.000726)	0.000711 (0.000155)
Head white (dummy)	0.000732 (0.000696)	0.00132 (0.000153)
Head female (dummy)	0.00000442 (0.000780)	0.000106 (0.000148)
Head college graduation (dummy)	0.00132 (0.00111)	0.00102 (0.000184)
Head HS graduate (dummy)	0.000377 (0.000724)	0.000541 (0.000144)
Head single (dummy)	0.0000791 (0.00125)	0.00437 (0.000238)
Head professional (dummy)	0.0000859 (0.000779)	0.000000675 (0.000132)
Head hours worked a year (dummy)	-0.00419 (0.00481)	0.0000326 (0.000794)
Spouse hours worked a year (dummy)	-0.000113 (0.00478)	0.00812 (0.00106)
Head nonworker (dummy)	0.00198 (0.00110)	0.000163 (0.000228)
Spouse nonworker (dummy)	0.000199 (0.00115)	0.00199 (0.000226)
Males ages 2 through 15	0.00116 (0.000423)	-0.000214 (0.000124)
Females ages 2 through 15	0.000617 (0.000374)	-0.000166 (0.000125)
Northeast (dummy)	0.00320 (0.00146)	0.00318 (0.000233)
North Central (dummy)	0.00185 (0.00107)	0.00147 (0.000185)
South (dummy)	0.00222 (0.00110)	0.00168 (0.000191)
West (dummy)	0.00318 (0.00106)	0.00256 (0.000201)

(continued)

Table 2. Continued

Coefficient	Estimates (standard errors)	
	Interior solution model	Boundary solution model
<i>Long-distance share equation</i>		
$\eta_{DK}, k = 1, 2, \dots, K$		
Age of head $\times 10^{-2}$	0.0385 (0.0125)	0.0115 (0.00391)
(Age of head) ² $\times 10^{-4}$	-0.0157 (0.0123)	-0.00535 (0.00429)
Number in household	-0.00311 (0.000719)	-0.000180 (0.000125)
Members ≥ 65 years old	-0.000313 (0.000839)	0.000439 (0.000287)
Head white (dummy)	-0.000193 (0.00109)	0.000765 (0.000289)
Head female (dummy)	-0.00292 (0.00120)	-0.000847 (0.000265)
Head college graduate (dummy)	-0.00329 (0.00174)	-0.00125 (0.000340)
Head HS graduate (dummy)	-0.000393 (0.000893)	-0.00000832 (0.000275)
Head single (dummy)	-0.00141 (0.00200)	0.00325 (0.000428)
Head professional (dummy)	0.000657 (0.00116)	0.0000482 (0.000247)
Head hours worked a year	0.00962 (0.00708)	0.000329 (0.0000146)
Spouse hours worked a year	0.00992 (0.00718)	0.0102 (0.00183)
Head nonworker (dummy)	0.00555 (0.00182)	-0.0000313 (0.000429)
Spouse nonworker (dummy)	0.0000657 (0.00171)	0.00164 (0.000386)
Males ages 2 through 15	0.00494 (0.00104)	0.000515 (0.000208)
Females ages 2 through 15	0.00388 (0.000884)	0.000891 (0.000215)
Northeast (dummy)	0.0106 (0.00216)	0.00442 (0.000413)
North Central (dummy)	0.00938 (0.00171)	0.00418 (0.000315)
South (dummy)	0.00768 (0.00169)	0.00284 (0.000315)
West (dummy)	0.00296 (0.00179)	0.00212 (0.000321)

(continued)

Table 2. Continued

Coefficient	Estimates (standard errors)	
	Interior solution model	Boundary solution model
<i>Food share equation</i>		
$\eta_{Fk}, k = 1, 2, \dots, K$		
Age of head $\times 10^{-2}$	-0.00121 (0.149)	0.348 (0.0207)
(Age of head) ² $\times 10^{-4}$	-0.00822 (0.144)	-0.347 (0.0221)
Number in household	-0.0194 (0.00540)	0.00427 (0.000710)
Members ≥ 65 years old	-0.0135 (0.0122)	0.00519 (0.00160)
Head white (dummy)	-0.0388 (0.0118)	-0.00487 (0.00173)
Head female (dummy)	0.0356 (0.0131)	0.0137 (0.00155)
Head college graduate (dummy)	0.0356 (0.0179)	0.0220 (0.00192)
Head HS graduate (dummy)	0.0147 (0.0107)	0.0102 (0.00152)
Head single (dummy)	-0.00543 (0.0245)	0.0682 (0.00228)
Head professional (dummy)	0.00618 (0.0136)	0.00394 (0.00135)
Head hours worked a year	0.00760 (0.0779)	0.0439 (0.00794)
Spouse hours worked a year	-0.0113 (0.0974)	0.129 (0.00982)
Head nonworker (dummy)	0.0307 (0.0173)	0.00731 (0.00179)
Spouse nonworker (dummy)	-0.0134 (0.0240)	0.0250 (0.00207)
Males ages 2 through 15	0.0104 (0.00860)	-0.00957 (0.00119)
Females ages 2 through 15	0.000764 (0.00632)	-0.00696 (0.00127)
Northeast (dummy)	-0.0181 (0.0293)	0.0246 (0.00226)
North Central (dummy)	0.0158 (0.0182)	0.0190 (0.00182)
South (dummy)	0.0295 (0.0177)	0.0272 (0.00185)
West (dummy)	-0.0246 (0.0220)	0.00836 (0.00194)

(continued)

Table 2. Continued

Coefficient	Estimates (standard errors)	
	Interior solution model	Boundary solution model
<i>Clothing share equation</i>		
$\eta_{ck}, k = 1, 2, \dots, K$		
Age of head $\times 10^{-2}$	0.216 (0.0344)	0.174 (0.0161)
(Age of head) ² $\times 10^{-4}$	-0.164 (0.0316)	-0.138 (0.0176)
Number in household	0.00194 (0.00125)	0.00459 (0.000517)
Members ≥ 65 years old	-0.000607 (0.00263)	0.000107 (0.00112)
Head white (dummy)	0.00546 (0.00278)	0.00714 (0.00117)
Head female (dummy)	-0.0205 (0.00440)	-0.00953 (0.00106)
Head college graduate (dummy)	-0.00465 (0.00527)	-0.000484 (0.00133)
Head HS graduate (dummy)	0.0000472 (0.00252)	0.000387 (0.00113)
Head single (dummy)	0.0148 (0.00639)	0.0284 (0.00168)
Head professional (dummy)	-0.0137 (0.00433)	-0.00693 (0.000990)
Head hours worked a year	-0.0274 (0.0245)	-0.0171 (0.00566)
Spouse hours worked a year	0.0342 (0.0280)	0.0641 (0.00703)
Head nonworker (dummy)	0.00264 (0.00480)	-0.00470 (0.00176)
Spouse nonworker (dummy)	0.00910 (0.00626)	0.0175 (0.00158)
Males ages 2 through 15	0.00468 (0.00235)	-0.00144 (0.000863)
Females ages 2 through 15	-0.00180 (0.00211)	-0.00306 (0.000834)
Northeast (dummy)	-0.00977 (0.00715)	0.0000689 (0.00163)
North Central (dummy)	-0.00719 (0.00471)	-0.00288 (0.00132)
South (dummy)	0.000360 (0.00474)	0.00294 (0.00133)
West (dummy)	0.000360 (0.00474)	0.00271 (0.00143)

(continued)

Table 2. Continued

<i>Coefficient</i>	<i>Estimates (standard errors)</i>	
	<i>Interior solution model</i>	<i>Boundary solution model</i>
<i>Other share equation</i>		
$\eta_{ok}, k = 1, 2, \dots, K$		
Age of head $\times 10^{-2}$	0.116 (0.173)	0.630 (0.0233)
(Age of head) ² $\times 10^{-4}$	-0.212 (0.165)	-0.661 (0.0249)
Number in household	0.00323 (0.00485)	0.0192 (0.000809)
Member ≥ 65 years old	-0.0189 (0.0159)	0.00642 (0.00178)
Head white (dummy)	-0.0526 (0.0147)	-0.00484 (0.00188)
Head female (dummy)	-0.00298 (0.0158)	-0.00169 (0.00168)
Head college graduate (dummy)	0.0138 (0.0243)	0.0200 (0.00212)
Head HS graduate (dummy)	-0.00938 (0.0133)	0.00273 (0.00170)
Head single (dummy)	0.0445 (0.0304)	0.130 (0.00273)
Head professional (dummy)	0.00779 (0.0178)	0.00272 (0.00153)
Head hours worked a year	0.0422 (0.101)	0.0294 (0.00914)
Spouse hours worked a year	0.0630 (0.133)	0.277 (0.0118)
Head nonworker (dummy)	0.0712 (0.0223)	-0.00236 (0.00251)
Spouse nonworker (dummy)	0.0253 (0.0301)	0.0684 (0.00271)
Males ages 2 through 15	0.0219 (0.00968)	-0.00830 (0.00139)
Females ages 2 through 15	0.00590 (0.00761)	-0.00812 (0.00139)
Northeast (dummy)	0.0381 (0.0368)	0.0521 (0.00250)
North Central (dummy)	0.0577 (0.0244)	0.0409 (0.00203)
South (dummy)	0.0840 (0.0246)	0.0527 (0.00208)
West (dummy)	0.0511 (0.0260)	0.0445 (0.00215)

(continued)

Table 2. Continued

Coefficient	Estimates (standard errors)	
	Interior solution model	Boundary solution model
<i>Price heterogeneity probability and support point estimates</i>		
π_1 , Urban		0.0466 (0.00252)
π_2 , Rural		0.0452 (0.00530)
θ_1 , Urban		1.05 (0.00271)
θ_1 , Rural		1.05 (0.00560)
θ_2 , Urban		0.0633 (0.0181)
θ_2 , Rural		0.0421 (0.0139)
γ_1 , Urban		1.05 (0.00275)
γ_1 , Rural		1.05 (0.00574)
γ_2 , Urban		0.0196 (0.00875)
γ_2 , Rural		0.0150 (0.00736)

Source: Author's calculations. *L* = local service, *D* = long-distance service, *F* = food, *C* = clothing, *O* = other, and *M* = total expenditure.

coefficient estimates for the vast majority of demographic variables imply that these variables substantially improve the explanatory power of both demand systems. A Wald test of the null hypothesis that all demographic variables do not enter any of the share equations is overwhelmingly rejected for both models.

Table 3 reports the Lagrange Multiplier statistics against heteroskedasticity of the elements of ϵ conditional on the log of prices and log total expenditure for all five share equations for the interior solution model. For all share equations the test statistic is substantially larger than the critical value for any conventional size hypothesis test. This result provides strong evidence against homoskedastic error variances relative to the alternative hypothesis of heteroskedasticity conditional on log prices and log total expenditure; that finding is implied by my

Table 3. Share Disturbance Heteroskedasticity Test, Lagrange Multiplier Test Statistics for Interior Solution Model

<i>Share equation</i>	<i>Model</i>
Local expenditure	268.38
Long-distance expenditure	101.94
Food expenditure	288.16
Clothing expenditure	164.80
Other nondurable expenditure	288.08

Source: Author's calculations.

Note: $\chi^2_{27} (\alpha = 0.05) = 40.11$, $\chi^2_{27} (\alpha = 0.01) = 46.96$.

approach to including ϵ as an unobservable vector of household-level variables in the indirect utility function in a manner consistent with utility-maximizing behavior.

For this reason, the standard errors given in table 2 are computed from the heteroskedasticity-consistent quasi-maximum likelihood covariance matrix estimates given by White.²⁹ Because of the presence of heteroskedasticity, comparing values of the quasi-maximum likelihood objective function cannot be used to compute valid test statistics, so all hypotheses about the interior solution model are examined using Wald statistics based on this covariance matrix estimate. I use the inverse of the matrix of outer products of the gradients of the individual terms of the log-likelihood function to compute standard error estimates and as the covariance matrix used to construct all Wald tests for the boundary solution estimates.

I next test for homothetic separability of local and long-distance phone service from food, clothing, and other nondurable goods in the household-level direct utility function for the translog model. This test asks whether the household-level direct utility function, $U(x_1, x_2, \dots, x_N)$, can be written as $U[g(x_1, x_2), x_3, \dots, x_N]$, where x_i is the quantity consumed of good i and $g(\cdot, \cdot)$ is a homothetic aggregator function. Assuming x_1 and x_2 are the amounts of local and long-distance service consumed, $g(x_1, x_2)$ then represents the telephone service aggregate good.

If this restriction on the household-level direct utility function holds, then the household's demand problem can be thought of in the two-stage budgeting context, where the household first determines its demands for the telephone aggregate and the other three goods using a price index for this telephone aggregate and the prices of the other

29. White (1982).

goods; then conditional on the total amount of telephone expenditures determined from this first-stage problem, the household decides the optimal allocation of spending between local and long-distance service by maximizing $g(x_1, x_2)$ subject to $p_1 x_1 + p_2 x_2 = M_T$, where M_T is total telephone expenditures determined in the first-stage optimization problem.

Homothetic separability of the household-level direct utility function is implicit in any analysis of the demand for local and long-distance service that ignores the household's expenditures on, or the prices for, all other goods consumed. Without this restriction on its utility function, a household cannot solve for the optimal local versus long-distance split without regard to its optimal demand for all other goods. Almost all analyses of telecommunications demand make this two-stage budgeting assumption. My multigood household-level dataset provides an ideal opportunity to investigate its empirical validity.³⁰

Although the restriction of homothetic separability given above is specified in terms of the household's direct utility function, theorem 4.4 of Blackorby, Primont, and Russell shows that homothetic separability of the direct utility function is equivalent to homothetic separability of the indirect utility function.³¹ In terms of the parameters of the translog function, global imposition (for all values of prices and total expenditure) of this restriction for a household with attribute vector A implies

$$(12) \quad \frac{\alpha_1^*(A)}{\alpha_2^*(A)} = \frac{\gamma_{1i}}{\gamma_{2i}} \text{ for } i = 1, 2, \dots, N,$$

where $\alpha_i^*(A)$ is defined in equation A-1 in appendix 1, the subscripts ¹ and ² denote local and long-distance service, and $N = 5$.

This test involves five nonlinear restrictions on the parameters of the translog demand system. Because this test depends on specific attributes of each household through the ratio $\alpha_1^*(A)/\alpha_2^*(A)$, ideally I should investigate its validity for each household. For each observation I compute the Wald test statistic for the five nonlinear restrictions given in equation 12 for both sets of parameter estimates. For the interior solution estimates the smallest value of the Wald statistic over all obser-

30. See Taylor (1994) for more evidence supporting the importance of examining the validity of this restriction on household preferences.

31. Blackorby, Primont, and Russell (1978).

vations in the sample is 72.36 and the largest is 76.99. For the boundary solution model the smallest value of the Wald statistic is 51.86 and the largest value is 218.65. These results provide substantial evidence against the validity of equation 12 for both model estimates.

An alternative way to investigate the equation's validity that does not involve computing test statistics for all observations in the sample is to examine the validity of the four nonlinear restrictions in the equation that do not depend on A . The Wald test for these four restrictions is 72.35 for the interior solution model and 45.57 for the boundary solution model, providing significant evidence against the last four nonlinear constraints given in equation 12.

Finally, I investigate whether this restriction holds when the first ratio in equation 12 is evaluated at the sample mean of A . In this case the interior solution Wald statistic is 72.52 and the boundary solution Wald statistic 255.41, which provides more evidence against the validity of these nonlinear restrictions. Taken together all of these results provide strong evidence against the validity of homothetic separability of the household's utility function in local and long-distance service. Remember, however, that this rejection of homothetic separability, and therefore the validity of the two-stage budgeting assumption, is conditional on the maintained hypothesis of a translog indirect utility function. In addition, imposing homothetic separability on the translog function destroys the second-order flexibility property of this function.³² Unfortunately, it also destroys the second-order flexibility of all other existing flexible functional forms that have been used to test for this restriction on utility functions. Although I can reject homothetic separability conditional on the assumed translog indirect utility function, I do not know if this is because the true underlying utility function has been incorrectly specified or because the homothetic separability null hypothesis is false.

Tests for additive separability of a household's indirect utility function for the interior and boundary solution models are also overwhelmingly rejected for both models. Additive separability implies the following six restrictions on the second-order parameters of the translog indirect utility function:

$$\beta_{lf} = 0, \beta_{lc} = 0, \beta_{lo} = 0, \beta_{df} = 0, \beta_{dc} = 0, \beta_{do} = 0,$$

32. See Blackorby, Primont, and Russell (1977) for a discussion of this point.

where l = local, d = long-distance, f = food, c = clothing and o = other nondurable goods. The Wald statistic is 10,777.0 for the interior model and 143.7 for the boundary solution model. Given these separability results, for both models I use the demand system estimates that do not impose any separability restrictions to perform the welfare calculations.

Assessing the Welfare Effects of Price Changes

Using the two integrable demand systems, I now assess the impact of various price-change scenarios for local and long-distance service on household-level welfare. I consider six scenarios. The first two involve increases in the price of local service alone. The second two involve price increases in local service accompanied by equivalent percentage decreases in the price of long-distance service. The final two involve percentage decreases in the price of long-distance service that are twice as large as the price increases in local service.

The six scenarios are

- A 20 percent increase in the price of local service alone,
- A 40 percent increase in the price of local service alone,
- A 20 percent increase in the price of local service accompanied by a 20 percent decrease in the price of long-distance service,
- A 40 percent increase in the price of local service accompanied by a 40 percent decrease in the price long-distance service,
- A 10 percent increase in the price of local service combined with a 20 percent decrease in the price of long-distance service, and
- A 20 percent increase in the price of local service combined with a 40 percent decrease in the price of long-distance service.

The first two scenarios attempt to capture the range of likely effects from increasing the price of local service. The last four attempt to assess the likely impacts of balancing a local service price increase with a corresponding decrease in the price of long-distance service—the scenario that might be expected if long-distance access charges and local

Table 4. Own-Price and Expenditure Elasticity Estimates

<i>Elasticity</i>	<i>Interior solution model</i>			<i>Boundary solution model</i>		
	<i>5th percentile</i>	<i>Mean</i>	<i>95th percentile</i>	<i>5th percentile</i>	<i>Mean</i>	<i>95th percentile</i>
$e_{L,L}$	-0.96	-0.88	-0.79	-0.54	-0.39	-0.26
$e_{D,D}$	-3.02	-2.07	-1.62	-2.38	-1.80	-1.50
$e_{F,F}$	-1.49	-1.35	-1.27	-0.83	-0.76	-0.63
$e_{C,C}$	-4.99	-2.85	-1.70	-7.76	-3.53	-1.51
$e_{O,O}$	-1.03	-1.00	-0.98	-0.91	-0.88	-0.82
$e_{M,L}$	-0.28	0.11	0.33	0.11	0.30	0.45
$e_{D,D}$	0.63	0.75	0.82	0.45	0.65	0.77
$e_{M,F}$	0.74	0.84	0.89	0.68	0.81	0.89
$e_{M,C}$	1.12	1.61	2.51	1.18	2.68	5.86
$e_{M,O}$	1.06	1.11	1.22	1.05	1.12	1.29

Source: Author's calculations.

Note: L = local service, D = long-distance service, F = food, C = clothing, O = other nondurables, and M = total expenditure.

service prices were to be rebalanced to leave local exchange carrier revenues essentially unchanged.³³

This framework also allows me to assess the regressivity of these proposed price increases as well as to determine which types of households, as measured by their observable characteristics, would bear a greater portion of the burden of these price changes. Finally, I can use the CES weights to extrapolate the household-level welfare change results to the U.S. population at large and to determine whether these price-change scenarios result in a net welfare gain or loss for the population of U.S. households.

First, however, I examine the sample average own-price and total expenditure elasticity estimates given in table 4 for the interior and boundary solution models. With some important exceptions, the household-level mean elasticity estimates are similar across the two indirect utility functions. Consistent with figure 2, the total expenditure elasticities for local service are very small. The boundary solution model recovers substantially smaller absolute value own-price elasticities for local service and slightly smaller values for long-distance service than

33. Appendix 2 sets out a framework for determining revenue-neutral price-change combinations using estimates of the revenue shares of the local carriers' three major products—local service, network access service, and long-distance service—and estimates of the U.S. population own- and cross-price demand elasticities for local and long-distance telephone service from the household sector.

those produced by the interior solution model. For both models, the mean expenditure elasticity of local service is slightly positive, with the boundary model value slightly larger. At the mean values for the sample, I find a price-elastic demand for long-distance service (with the boundary model less elastic) and an expenditure elasticity that is greater than the one for local service (with this statement less true for the boundary solution model). Both mean expenditure elasticities indicate that long-distance service is a normal, not a luxury, good.

A major difference is that clothing is much more price elastic in the boundary model than in the interior model. Recall that there are many joint zeros for clothing and long-distance phone service, so it is no surprise that the models recover different estimates of the price effects involving these two goods. For both models the range of the price elasticity of clothing indicates the potential for substantial differences in the welfare and consumption impacts of price changes across households. These differences in price effects across the two models combine to result in the opposite aggregate welfare conclusions for the two equal price-change scenarios.

The aggregate own- and cross-price elasticity estimates implied by both the boundary and interior solution models, combined with the most recent values for the national average of local exchange carrier revenue shares by product, imply that the price changes that balance a price increase for local service with an equal percentage decrease in long-distance service would leave aggregate national revenues to the local carriers from the household sector unchanged. I nonetheless also consider price-change scenarios where the long-distance price decrease is twice the local service price increase because there are plausible values for the aggregate household sector elasticities that indicate such price changes would leave aggregate revenues from the household sector unchanged.

For the welfare analysis I compute the compensating variation associated with each of these six price changes. Assuming that P^0 is the initial vector of prices, M^0 is initial total expenditure, and P^1 is the proposed vector of prices, $V(P, M, A)$ is the indirect utility for a household facing prices P , and with total expenditure M and characteristics A , the compensating variation is the value of CV that solves the nonlinear equation

$$V(P^1, M^0 + CV, A) = V(P^0, M^0, A).$$

In other words, CV is the amount of additional total expenditure that must be given to a household with characteristics A for it to be indifferent between total expenditure $M^0 + CV$ and price P^1 and total expenditure M^0 and price P^0 . A negative value of CV implies that the new prices are welfare enhancing.

Both the boundary and interior solution models assume that the disturbances to the demand system are unobservable household characteristics in the indirect utility function $V(P, M, A, \nu)$, where ν is the vector of disturbances to the demand system. As emphasized in the discussion on equations 4 and 5, I do not assume a specific functional form for how these disturbances enter the indirect utility function for the interior model. To compute the likelihood function for the boundary model, a specific functional form must be assumed for the dependence of $V(P, M, A, \nu)$ on ν . Computing the compensating variation under this interpretation of the disturbances to the demand system implies that CV is a function of ν , because it is the solution to the equation $V(P^1, M^0 + CV, A, \nu) = V(P^0, M^0, A, \nu)$. For each household I can compute $E[CV(\nu)]$, the expected value of its compensating variation with respect to the distribution of unobserved household characteristics, and $\text{Var}[CV(\nu)]$, the variance of its compensating variation. For the boundary solution model, computing $CV(\nu)$ is straightforward for any value of ν , because the dependence of $V(P, M, A, \nu)$ on ν is specified in equation 9, so that taking a large number of draws from the estimated distribution of ν for each household and computing the sample moments of $CV(\nu)$ for these draws yields simulation estimates of $E[CV(\nu)]$ and $\text{Var}[CV(\nu)]$. For the interior solution model, the dependence of indirect utility function on the vector of unobserved household characteristics is left unspecified, so the expected value and variance of CV for each household cannot be computed without further assumptions. Because it is unclear how the unobserved household characteristics should enter the indirect utility function for the interior solution model, I decided instead to compute the compensating variation for each household for both the boundary and interior model estimates at the expected value of the disturbances to the demand system, so the vector of unobserved household characteristics is set equal to zero for all compensating variation calculations.

Therefore, any differences in the welfare calculations across the interior and boundary solution models are attributable to the differences in the parameter estimates of the indirect utility function across the two models rather than to differences in the estimated distribution of unobservable household characteristics.

As a check of the difference between $CV(v=0)$ and $E[CV(v)]$, I computed both magnitudes for the boundary solution model. For all of the price-change scenarios considered, the difference between $CV(v=0)$ and $E[CV(v)]$ is minor because of the very small estimated variances of the elements of v . At most these two magnitudes differ only in the second significant digit. Although $\text{Var}[CV(v)]$ reveals significant uncertainty in $CV(v)$ for any specific household, computing either the mean of $CV(v)$ over all households in my sample or the estimated U.S. population mean of $CV(v)$ using the CES weights yields standard errors that are less than 1 percent of these estimated means. These standard errors are computed using the estimated values of $\text{Var}[CV(v)]$ for each household and assuming that the v s are independent and identically distributed across households. Consequently, there is no difference in the quantitative conclusions that I am able to draw from the welfare analysis of the boundary solution model from setting the value of the vector of unobserved household characteristics equal to its mean value in computing the distribution of the welfare impacts of the price-change scenarios I consider.

Using duality theory, I know that associated with $V(P, M, A)$ is the expenditure function $E(P, U, A)$, which gives the minimum expenditure level necessary to achieve utility level U . Define $U^0 = V(P^0, M^0, A)$ as the utility at existing prices and total expenditure. By the definition of the expenditure function, $M^0 = E(P^0, U^0, A)$, and by the definition of compensating variation, $M^0 + CV = E(P^1, U^0, A)$. I can then define the utility-constant or percentage true cost-of-living increase as a result of increasing prices from P^0 to P^1 as

PTCLI = Percentage True Cost-of-Living Increase

$$= 100 \times \left(\frac{E(P^1, U^0, A)}{E(P^0, U^0, A)} - 1 \right) = 100 \times \left(\frac{CV}{M^0} \right).$$

This true cost-of-living increase can be computed for all observations

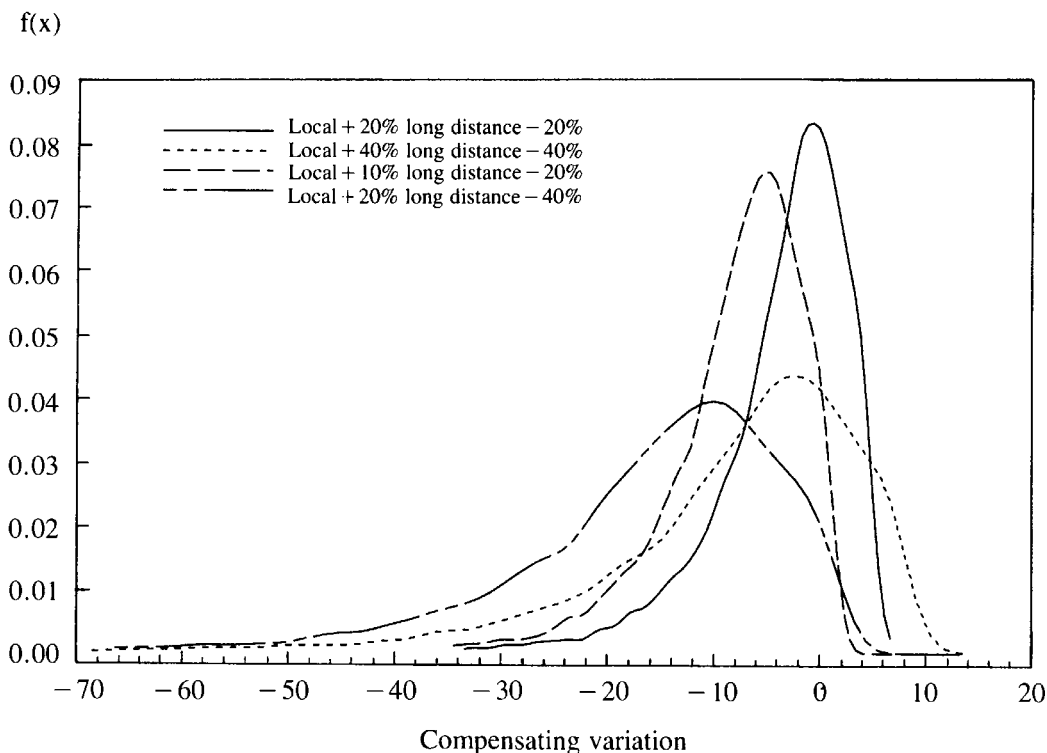
in the sample. Examining how this ratio changes with M^0 allows me to determine the extent of the regressivity or progressivity of these price-change scenarios. Finally, to assess how the true cost-of-living increase relates to the characteristics of the household, I estimate best linear predictor functions for the percentage true cost-of-living increase as a function of household characteristics and total expenditure.

For all of the scenarios to yield theoretically valid household-level compensating variations, the estimated demand system must satisfy all the restrictions implied by utility-maximizing behavior at the prices, total expenditures, and household characteristics for that observation. Because both models are estimated with summability, homogeneity, and symmetry imposed, I need only check that the negative semi-definiteness of the Slutsky matrix holds at each observation. This process involves computing the Slutsky matrix given in equation 3, with $D(P, M, A)$ in place of $D(P, M)$, for each observation and then computing the five eigenvalues of this matrix and verifying that they are nonpositive. As discussed earlier, because there are no necessary and sufficient conditions (there are sufficient conditions) on the parameters of the translog indirect utility function that guarantees the Slutsky matrix is negative semi-definite for all prices and expenditures, I must follow this procedure to select those observations that can be used to compute the household-level welfare changes that result from the price-change scenarios.

For the interior solution model I lose only a small percentage of observations due to failure of the negative semi-definiteness of the Slutsky matrix. Out of the full sample of 11,467 observations, 11,344 satisfy the necessary curvature restrictions. All of the calculations for the interior solution model are based on this subsample. For the boundary solution model I lose more observations. In this case, 8,492 of the observations satisfy the negative semi-definiteness of the Slutsky matrix, and all calculations for this model are for this restricted sample.³⁴ For both models those observations that fail the restriction have just a single marginally positive eigenvalue that is a small fraction of the smallest, in absolute value, negative eigenvalue.

34. Linear probability models predicting whether individual observations satisfy these curvature restrictions as a function of observable household-level characteristics, including total expenditure, did not uncover any readily discernable reasons why observations failed to satisfy these restrictions.

Figure 6. Densities of Sample Household-Level Compensating Variations in January 1988 Dollars for Price-Change Scenarios for Interior Model Estimates

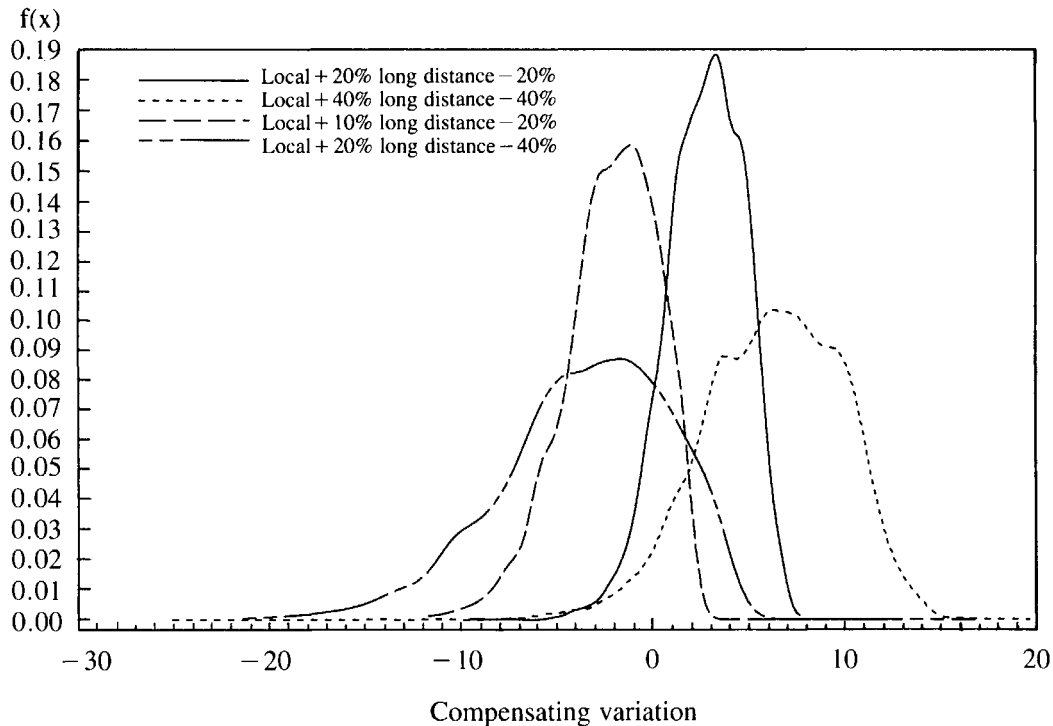


Source: Author's calculations.

Figure 6 plots the sample kernel density of household-level compensating variation in January 1988 dollars for the four two-price-change scenarios for the interior solution estimates. Figure 7 plots these same magnitudes for the boundary solution estimates. The interior solution estimates show considerably more variability and tend to be more negative than the boundary solution estimates, indicating that the interior solution model finds all of the two-price-change scenarios substantially more welfare-enhancing than does the boundary solution model. The summary statistics associated with these densities presented in table 5 bear out this point.

The welfare implications across the two models diverge for the price-change scenarios that set the local price increase equal to the long-distance price decrease. As shown in appendix 2, for the aggregate household demand elasticity estimates implied by the two models, these two-price-change scenarios are consistent with keeping local exchange carrier revenue from the U.S. household sector neutral. The interior

Figure 7. Densities of Sample Household-Level Compensating Variations in January 1988 Dollars for Price-Change Scenarios for Boundary Model Estimates



Source: Author's calculations.

solution model estimates negative sample mean compensating variations for the price-change scenarios that are equal but opposite in sign: $-\$3.48$ for a price change in each service of 20 percent and $-\$7.95$ for a price change in each service of 40 percent, indicating that the average household in the sample receives welfare gains from these price changes.

Conversely, the boundary solution finds mean compensating variations for these two scenarios of $\$2.66$ and $\$6.14$, respectively, implying a sample average household-level welfare loss. For the scenarios where the percentage price decrease for long-distance is double the increase for local service, both models find mean welfare gains (negative sample mean household-level compensating variations), although the 95th percentile values for the boundary model imply that some households experience welfare losses associated with these price changes.

The more pessimistic view of the combination price changes emerging from the boundary model stems from the significantly higher esti-

Table 5. Compensating Variations for Price-Change Scenarios

In January 1988 dollars

Scenario	Household-level sample distribution			Estimated U.S. population	
	5th percentile	Mean	95th percentile	Mean	Median
<i>Interior solution model</i>					
$P_L + 20\%; P_D + 0\%$	6.98	9.45	12.12	9.43	9.39
$P_L + 40\%; P_D + 0\%$	13.01	17.59	22.61	17.55	17.49
$P_L + 20\%; P_D - 20\%$	-15.94	-3.48	4.04	-2.95	-1.99
$P_L + 40\%; P_D - 20\%$	-32.12	-7.95	6.52	-6.91	-5.06
$P_L + 10\%; P_D - 20\%$	-20.78	-7.96	0.19	-7.41	-6.56
$P_L + 20\%; P_D - 40\%$	-40.56	-15.91	-0.29	-14.85	-13.18
<i>Boundary solution model</i>					
$P_L + 20\%; P_D + 0\%$	6.82	9.26	11.91	9.24	9.17
$P_L + 40\%; P_D + 0\%$	13.06	17.86	23.12	17.83	17.68
$P_L + 20\%; P_D - 20\%$	-0.80	2.66	5.60	2.61	2.81
$P_L + 40\%; P_D - 40\%$	0.09	6.14	11.46	6.03	6.34
$P_L + 10\%; P_D - 20\%$	-6.52	-2.23	1.28	-2.27	-1.97
$P_L + 20\%; P_D - 40\%$	-11.37	-3.60	2.78	-3.68	-3.13

Source: Author's calculations.

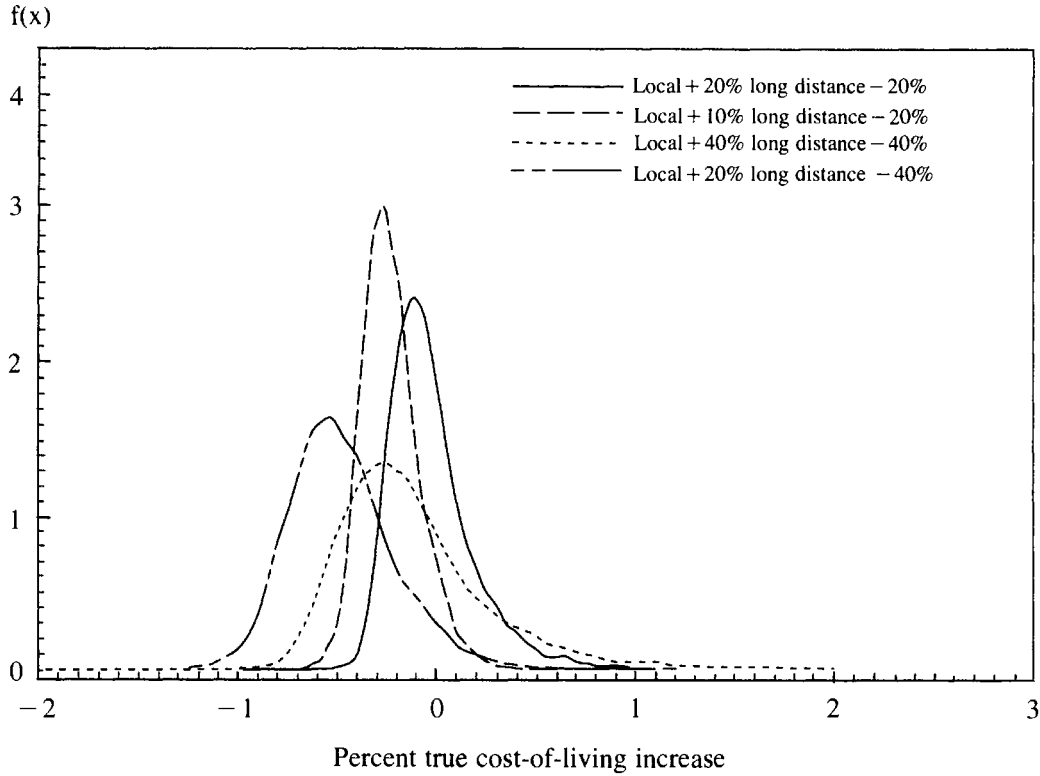
Note: P_L = price of local service; P_D = price of long-distance service.

mates of the expenditure and price elasticity for clothing under that model compared with the interior solution model. In addition, I also estimate a substantially higher cross-price elasticity between clothing and long-distance service under the boundary solution model. These elasticity estimates imply that instead of buying more long-distance service as the price decreases, many households buy more clothing (the interior solution model predicts that decreases in the price of long-distance service result in the purchase of relatively more long-distance service).

Consequently, under the boundary solution model the net effect of these price changes for these households is still a welfare loss. This effect is particularly true for the households with low total expenditures that consume very small amounts of long-distance service initially. Because it explicitly accounts for the existence of corner solutions in the household's choice problem, the boundary solution model is ideally suited for modeling the responses of these households.

Table 5 also shows the mean and median estimates of household-level compensating variations for the U.S. population associated with

Figure 8. Densities of Sample Household-Level Percent True Cost-of-Living Increases for Price-Change Scenarios for Interior Model Estimates

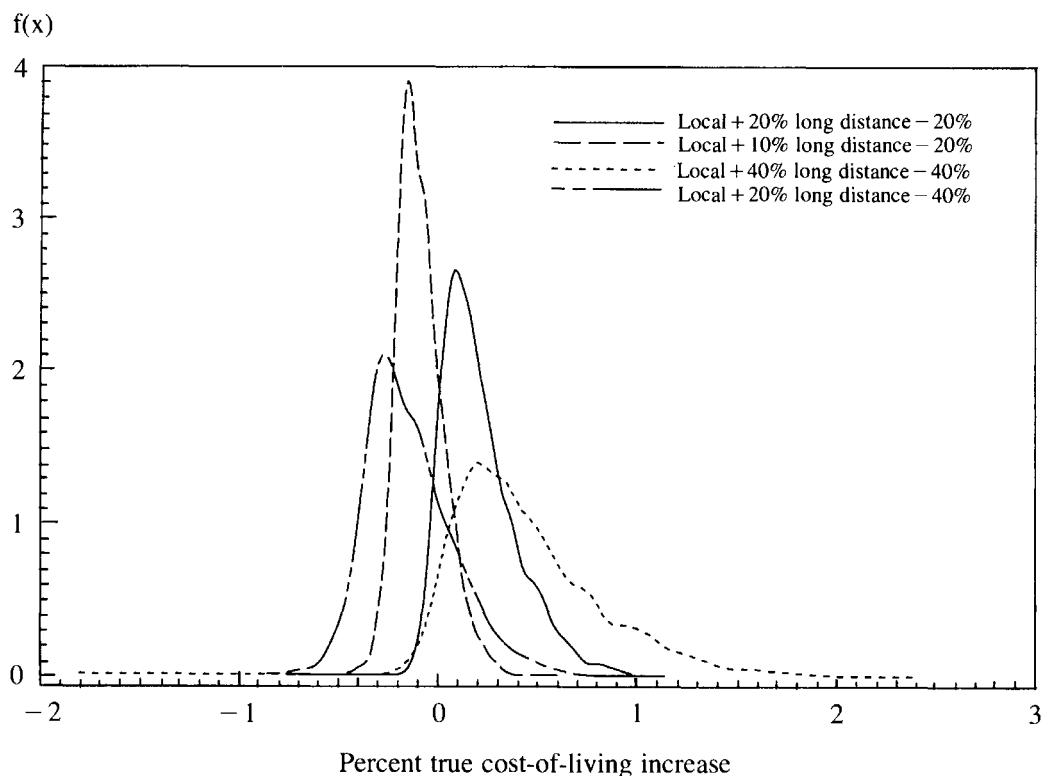


Source: Author's calculations.

these price-change scenarios. For all scenarios and both models, the sample mean is in close agreement with the national mean. Multiplying these mean per-household figures by the number of households in the United States yields an estimate of the net amount of money that could be raised (negative values) or must be given to households (positive values), assuming all U.S. households are paid their compensating variations, in order for all households to be indifferent between the existing price and the proposed prices. Both models rationalize the single price changes as aggregate welfare losers and the unequal price-change scenarios as net aggregate welfare gainers. However, the interior solution model finds the equal price-change scenarios to be net gainers, whereas the boundary solution finds them to be net losers for the national population of households.

Figures 8 and 9 plot the densities of the household-level percent true cost-of-living increases that result from the four two-price-change sce-

Figure 9. Densities of Sample Household-Level Percent True Cost-of-Living Increases for Price-Change Scenarios for Boundary Model Estimates



Source: Author's calculations.

narios. These densities show that for the same price-change scenario, the boundary model densities in figure 9 tend to shift rightward and be more concentrated than the comparable densities for the interior solution model in figure 8. This result implies that the boundary solution model finds, on average, a higher cost-of-living increase associated with each of the two-price-change scenarios than does the interior solution model.

Table 6 presents summary statistics on the household-level true cost-of-living increases for all of the welfare scenarios. The divergent welfare implications for the equal price-change scenarios continue for the true cost-of-living indexes, with the boundary model finding an increase on average and the interior model finding a decrease. Both models find a sample average true cost-of-living decrease associated with each of the unequal price-change scenarios.

The boundary solution estimates that the mean true cost-of-living

Table 6. True Cost-of-Living Increases for Price-Change Scenarios

Percent

Scenario	Household-level sample distribution			Estimated U.S. population	
	5th percentile	Mean	95th percentile	Mean	Median
<i>Interior solution model</i>					
$P_L + 20\%; P_D + 0\%$	0.16	0.42	0.84	0.43	0.38
$P_L + 40\%; P_D + 0\%$	0.29	0.78	1.56	0.81	0.70
$P_L + 20\%; P_D - 20\%$	0.30	-0.04	0.35	-0.03	-0.08
$P_L + 40\%; P_D - 40\%$	0.60	-0.14	0.56	-0.12	-0.20
$P_L + 10\%; P_D - 20\%$	0.45	-0.24	0.01	-0.24	-0.26
$P_L + 20\%; P_D - 40\%$	0.87	-0.49	-0.02	-0.48	-0.52
<i>Boundary solution model</i>					
$P_L + 20\%; P_D + 0\%$	0.33	0.56	0.98	0.56	0.51
$P_L + 40\%; P_D + 0\%$	0.65	1.08	1.87	1.07	0.98
$P_L + 20\%; P_D - 20\%$	-0.03	0.20	0.56	0.19	0.16
$P_L + 40\%; P_D - 40\%$	0.0	0.44	1.13	0.43	0.36
$P_L + 10\%; P_D - 20\%$	-0.26	-0.10	0.12	-0.10	-0.11
$P_L + 20\%; P_D - 40\%$	-0.44	-0.15	0.25	-0.15	-0.18

Source: Author's calculations.

Note: P_L = the price of local service; P_D = the price of long-distance service.

increase associated with a 40 percent increase in the price of local service only is 1.08 percent, with a 5th to 95th percentile range of 0.65 percent to 1.87 percent. The interior solution estimates find slightly smaller values. The largest increase over all households in the sample is 3.0 percent for the boundary solution estimates and 3.8 percent for the interior solution estimates. Based on these findings, neither model finds that a 40 percent increase in the price of local service results in a significant increase in the true cost-of-living for any of the households in the sample.

Table 6 also gives estimates for the average true cost-of-living increases associated with the six scenarios for the U.S. population as a whole. These means are very similar to the sample means and thus do not seem to represent a significant increase in the mean true cost of living. For the two equal price-change scenarios, the boundary solution model implies increases in the mean and median true cost of living for the national population, while the interior solution model implies decreases. Both models find the unequal price-change scenarios to lead to mean and median cost-of-living decreases for the population as a whole.

Are increases in the price of local phone service likely to cause some households to disconnect from the local network, as some observers argue? As I have discussed earlier, the interior solution model is poorly suited to addressing the problem of zero consumption, but the boundary solution model is designed with this aspect of household choice in mind. Conditional on a predicted positive expenditure for local service, I compute the number of negative predicted local demand shares as a result of the price changes in the six scenarios. I interpret a negative predicted share for local service as a disconnection from the telephone network. None of the scenarios predicts negative shares for local service for any household for either model, indicating that, at least for this sample of households, disconnection from the telephone network appears to be a very unlikely response.

To assess how the burden of these price changes is shared across households, I compute best linear predictor functions for the percent true cost-of-living increases (PTCLI) as a function of household characteristics and the log of total expenditure. The coefficients from these regressions provide an estimate of how the best linear predictor of PTCLI changes in response to any change in household characteristics or total expenditure. Table 7 presents these best linear predictor functions and White heteroskedasticity-consistent standard error estimates for the interior and boundary solution models for the scenario that increases local service by 20 percent and decreases long-distance service by 40 percent. The functions are very similar across the two models. The negative coefficient on the log of total expenditure indicates that households with higher total expenditures are predicted to have lower increases in their true cost of living. That means that the households with lower total expenditures bear a proportionately larger share of the price increases. Indeed, I estimate that the burden is steeply regressive—that is, that the burden increases as total household expenditures get lower. The boundary solution results show that the best linear predictor of PTCLI increases by approximately 1.4 percentage points for a 10 percent decrease in total expenditures. A plot of the true cost-of-living increase versus $\ln(M)$ reveals that this estimated relation is determined by the very regressive nature of these price changes at very low values of $\ln(M)$. At higher values of $\ln(M)$ these price changes are much less regressive.

Several other conclusions emerge from these regressions. Older

Table 7. Best Linear Predictors of Percent True Cost-of-Living Increase

<i>Variable</i>	<i>Estimated coefficient (standard error)</i>	
	<i>Interior solution model</i>	<i>Boundary solution model</i>
Constant	-0.297 (0.154)	0.921 (0.0335)
Age of head $\times 10^{-2}$	1.04 (0.0392)	-0.862 (0.0476)
(Age of head) ² $\times 10^{-4}$	0.0481 (0.0439)	1.49 (0.0503)
Number in household	-0.0769 (0.000949)	-0.0393 (0.00201)
Members ≥ 65 years old	-0.0177 (0.00217)	-0.0224 (0.00296)
Head white (dummy)	0.0291 (0.00284)	0.00405 (0.00366)
Head female (dummy)	-0.114 (0.00185)	-0.0576 (0.00385)
Head college graduate (dummy)	-0.188 (0.00244)	-0.176 (0.00434)
Head HS graduate (dummy)	-0.0411 (0.00200)	-0.0354 (0.00266)
Head single (dummy)	-0.0849 (0.00265)	-0.0698 (0.00638)
Head professional (dummy)	0.0232 (0.00140)	0.00438 (0.00262)
Head hours worked a year	0.508 (0.00984)	0.0523 (0.0171)
Spouse hours worked a year	0.329 (0.0100)	0.158 (0.0184)
Head nonworker (dummy)	0.138 (0.00321)	0.0219 (0.00510)
Spouse nonworker (dummy)	-0.0245 (0.00227)	-0.0450 (0.00413)
Males age 2 through 15	0.132 (0.00137)	0.0616 (0.00264)
Females age 2 through 15	0.125 (0.00137)	0.0876 (0.00274)
Northeast (dummy)	0.304 (0.00263)	0.186 (0.00814)
North Central (dummy)	0.287 (0.00201)	0.240 (0.00307)
South (dummy)	0.152 (0.00205)	0.0968 (0.00309)
West (dummy)	0.0575 (0.00206)	0.0115 (0.00390)
Log of nondurable expenditure	-0.0970 (0.00187)	-0.140 (0.00390)

Source: Author's calculations for a price-change scenario in which the price of local service rises 20 percent while the price of long-distance service drops 40 percent. The standard error estimates are from White (1980).

households, urban households, households with young children, and households with two working adults bear more than their proportionate share of the true cost-of-living increases. Repeating these best linear predictor calculations for other price-change scenarios yields similar conclusions about the relationship between true cost-of-living increases and household characteristics, including total expenditure.

These differences in true cost-of-living increases appear because household characteristics are allowed to shift the household-level indirect utility function and expenditure share equations, so that the price and expenditure elasticities differ across households according to these characteristics. Within my modeling framework, the variations in these true cost-of-living increases can only be explained in these terms. Because the household is the unit of analysis, I am unable to distinguish among the many within-household decisions that might explain these differences in price and expenditure responsiveness.

The Viability of Rate Rebalancing in Competitive Telecommunications Markets

Assuming the estimated demand systems are valid descriptions of the observed pattern of household-level consumption patterns, my calculations allow several conclusions to be drawn. All of the evidence seems consistent with the view that the vast majority of households will sustain very small welfare losses as a result of substantial increases in the price of local telephone service. Balancing these local service price increases with reductions in long-distance access charges seems likely to result in net welfare gains for many households.

According to my household-level demand function estimates, a combination of equal price increases for local service and price decreases for long-distance service are most likely to be revenue neutral, leaving total local exchange carrier revenues from the household sector unchanged. For these price-change scenarios, whether the sum of compensating variations for all U.S. households is positive or negative depends on how zeros are accounted for in the household's consumption choices. Accounting for zeros in a manner that acknowledges the existence of nonnegativity constraints in the household's utility-maximization problem shows that the sample and estimated U.S. pop-

ulation averages of these household-level compensating variations are slightly positive, implying an average household-level welfare loss. Neither model overturns the conventional belief that local price increases disproportionately burden low-income (in this case, low total-expenditure) households, older households, urban households, and households where both the head and spouse are employed. The boundary solution model, however, finds relatively larger welfare losses among low-income total expenditure households when price increases for local service are balanced with price decreases in long-distance service than does the interior solution model. According to the boundary solution model estimates, decreases in the price of long-distance have little benefit to low total expenditure households because they do not purchase much or any of this good, either at the original price or the new, lower price. Recall the discussion of figure 1. Despite these differences in aggregate welfare results, neither set of estimation results suggests that the burden is so great that households currently connected to the local network are likely to disconnect.

The separability test results signal the importance of modeling telephone demand jointly with the demand for all other goods in order to accurately measure price and expenditure elasticities and to perform theoretically valid welfare calculations, particularly for those households at the lower end of the nondurable expenditure (or income) distribution, which are more likely to have zero expenditures in one of the five goods at prevailing prices.

Finally, the comparison of the results from the two models emphasizes the importance of properly accounting for the presence of zeros in a household's consumption choice problem for the resulting average individual-level and aggregate net welfare calculations. This is particularly true for phone service, which makes up a small fraction of a household's budget.

Appendix 1

This appendix derives the likelihood function for the boundary solution model for three classes of observations depending on the number of zero expenditure shares: no zeros, one zero, and two zeros. Although the model can allow for up to $N-1$ zeros, where N is the number of

goods the household can potentially buy, I stop at the two-zero case because that is the maximum number of zeros in this sample. To simplify notation, let

$$(A-1) \quad \alpha_i^*(A) = (\alpha_i + \sum_{k=1}^K \eta_{ik} A_k) \text{ and}$$

$$DEN(P, M, A) = \sum_{i=1}^N \alpha_i^*(A) + \sum_{i=1}^N \sum_{j=1}^N \beta_{ij} \ln\left(\frac{p_j}{M}\right).$$

First consider the case of all nonzero expenditure shares. In terms of this notation

$$w_i = \frac{\alpha_i^*(A) + \sum_{k=1}^N \beta_{ik} \ln(p_k/M) + v_i}{DEN(P, M, A)},$$

so that

$$(A-2) \quad v_i = [DEN(P, M, A)] w_i - \left[\alpha_i^*(A) + \sum_{j=1}^N \beta_{ij} \ln(p_j/M) \right],$$

where w_i is the observed share of the i th good because all observed demands are positive. The likelihood function for observations with no zeros is

$$(A-3) \quad L_0(w|\Sigma, \Gamma, P, M, A) = (2\pi)^{-(N-1)/2} |D(P, M, A)|^{N-1} |\Sigma|^{-1/2} \exp[-1/2(v' \Sigma^{-1} v)]$$

where Γ is the vector comprising all of the parameters of the indirect utility function, and the elements of v are defined in equation A-2. The likelihood function for the no-zero case is very similar to the quasi-likelihood function given for the interior solution model, except that the change of variables from v_i to w_i yields a Jacobian of the transformation that is not equal to the identity matrix, because v_i is normalized by $D(P, M, A)$ in the share equation given in equation 10.

Consider the case where the virtual price is less than or equal to the actual price [$p_i^v(p) \leq p_i$] for one $i \in (1, 2, \dots, N)$. This event occurs if

$$(A-4) \quad \{[\alpha_i^*(A) + \sum_{k=1}^N \beta_{ik} \ln(p_k/M)] + v_i\} / \beta_{ii} \geq 0,$$

because $D(P, M, A)$ is less than zero, so this event is equivalent to the event

$$v_i/\beta_{ii} \geq - [\alpha_i^*(A) + \sum_{k=1}^N \beta_{ik} \ln(p_k/M)] / \beta_{ii} .$$

The remaining demands for the other goods can be computed by solving the household's budget-constrained utility maximization problem conditional on $w_i = 0$ with virtual prices in place of the actual prices for the goods not consumed, and the resulting demands are the same as those that result from the explicit solution of the Kuhn-Tucker conditions for the budget-constrained utility maximization problem, subject to the constraints that the household must consume nonnegative amounts of all goods. For good i , the zero consumption good, this virtual price, $p_i^*(p)$, is computed by solving equation A-4 for the value of p_i , which satisfies the inequality with equality. Under my functional form assumptions, the logarithm of the total expenditure normalized virtual price for the i th good is

$$\ln[\pi_i(P, M, A, v_i)] = - [\alpha_i^*(A) - \sum_{j=1, j \neq i}^N \beta_{ij} \ln(p_j/M) + v_i] / \beta_{ii} .$$

This is the virtual price that supports $w_i = 0$ and the other nonzero observed shares as the nonnegativity constrained utility-maximizing choices. The remaining shares are given by

$$w_j = \frac{NUM_j(P, M, A, v_i) + v_j}{DEN(P, M, A, v_i)}$$

where

$$NUM_j(P, M, A, v_i) = \alpha_j^*(A) + \sum_{k=1, k \neq i}^N \beta_{jk} \ln(p_k/M) + \beta_{ji} \ln[\pi_i(P, M, A, v_i)]$$

and

$$DEN(P, M, A, v_i) = \sum_{j=1}^N \alpha_j^*(A) + \sum_{j=1}^N \left\{ \sum_{k=1, k \neq i}^N \beta_{jk} \ln(v_k) + \beta_{ji} \ln[\pi_i(P, M, A, v_i)] \right\}$$

This implies

$$(A-5) \quad v_j = DEN(P, M, A, v_i) w_j - NUM_j(P, M, A, v_i).$$

Consequently, the likelihood function for this observation is

$$L_1(w|\Gamma, \Sigma, P, M, A) = \int_{-\infty}^{\infty} |DEN(P, M, A, v_i)|^{N-2} (2\pi)^{\frac{(N-1)}{2}} |\Sigma|^{-1/2} \\ \times \exp \left(-\frac{1}{2} v' \Sigma^{-1} v \right) dv_i \\ - [\alpha_i^*(A) + \sum_{k=1}^N \beta_{ik} \ln(p_k/M)]$$

where the remaining elements of the $(N-1)$ -dimensional vector v besides v_i are given in equation A-5. Because of this dependence of each of these v_j on v_i , this integral cannot be written as a multivariate normal probability. Consequently, general univariate numerical integration techniques must be employed to compute this likelihood function value.³⁵

Computing the likelihood function for the case of two zeros follows this same process. Suppose that $p_i^v(p) \leq p_i$ and $p_j^v(p) \leq p_j$ for two $i, j \in (1, 2, \dots, N)$. This event occurs if v_i and v_j satisfy the inequalities

$$\ln[\pi_i(P, M, A, v_i, v_j)] \leq \ln(p_i/M)$$

(A-6)

$$\ln[\pi_j(P, M, A, v_i, v_j)] \leq \ln(p_j/M)$$

where the right-hand sides of the inequalities in equation A-6, the expenditure normalized virtual prices for nonconsumed goods i and j , are given by

$$(A-7) \quad \begin{pmatrix} \ln[\pi_i(P, M, A, v_i, v_j)] \\ \ln[\pi_j(P, M, A, v_i, v_j)] \end{pmatrix} \\ = - \begin{pmatrix} \beta_{ii} & \beta_{ij} \\ \beta_{ji} & \beta_{jj} \end{pmatrix}^{-1} \begin{pmatrix} \alpha_i^*(A) + \sum_{m=1, m \neq i, j}^N \beta_{im} \ln(p_m/M) + v_i \\ \alpha_j^*(A) + \sum_{m=1, m \neq i, j}^N \beta_{jm} \ln(p_m/M) + v_j \end{pmatrix}.$$

35. I use an algorithm suggested by Gill and Miller (1972).

These are the virtual prices that support $w_i = 0$ and $w_j = 0$ and the remaining vector of positive shares as the solution to a budget-constrained utility maximization problem. The remaining positive shares are given by

$$(A-8) \quad w_k = \frac{NUM_k(P, M, A, v_i, v_j) + v_k}{DEN(P, M, A, v_i, v_j)}$$

where

$$\begin{aligned} NUM_k(P, M, A, v_i, v_j) = & \alpha_k^* + \beta_{ki} \ln[\pi_i(P, M, A, v_i, v_j)] \\ & + \beta_{kj} \ln[\pi_j(P, M, A, v_i, v_j)] \\ & + \sum_{m=1, m \neq i, j}^N \beta_{km} \ln(p_m/M) \end{aligned}$$

and

$$\begin{aligned} (A-9) \quad DEN(P, M, A, v_i, v_j) = & \sum_{k=1}^N \alpha_k^*(A) \\ & + \sum_{k=1}^N \left\{ \beta_{ki} \ln[\pi_i(P, M, A, v_i, v_j)] \right. \\ & + \beta_{kj} \ln[\pi_j(P, M, A, v_i, v_j)] \\ & \left. + \sum_{m=1, m \neq i, j}^N \beta_{km} \ln(p_m/M) \right\}. \end{aligned}$$

In this notation $v_k = DEN(P, M, A, v_i, v_j)w_k - NUM_k(P, M, A, v_i, v_j)$, so that the likelihood function for two zeros is the double integral

$$L_2(w|\Gamma, \Sigma, P, M, A)$$

$$\begin{aligned} = & \int_0^\infty \int_0^\infty |DEN(P, M, A, \tau_i, \tau_j)|^{N-3} \begin{vmatrix} \beta_{ii} & \beta_{ij} \\ \beta_{ji} & \beta_{jj} \end{vmatrix} (2\pi)^{(N-1)/2} |\Sigma|^{-1/2} \\ & \times \exp(-1/2 v' \Sigma^{-1} v) d\tau_i d\tau_j, \end{aligned}$$

where

$$\begin{pmatrix} \tau_i \\ \tau_j \end{pmatrix} = \begin{pmatrix} \ln(p_i/M) \\ \ln(p_j/M) \end{pmatrix} + \begin{pmatrix} \beta_{ii} & \beta_{ij} \\ \beta_{ji} & \beta_{jj} \end{pmatrix}^{-1} \\ \times \begin{pmatrix} \alpha_i^*(A) + \sum_{m=1, m \neq i,j}^N \beta_{im} \ln(p_m/M) + v_i \\ \alpha_j^*(A) + \sum_{m=1, m \neq i,j}^N \beta_{jm} \ln(p_m/M) + v_j \end{pmatrix}.$$

For τ_i and τ_j , the elements of v are defined as follows: the share equation disturbances for the goods with zero demands are

$$\begin{pmatrix} v_i \\ v_j \end{pmatrix} = \begin{pmatrix} \beta_{ii} & \beta_{ij} \\ \beta_{ji} & \beta_{jj} \end{pmatrix} \begin{pmatrix} \tau_i \\ \tau_j \end{pmatrix} - \begin{pmatrix} \alpha_i^*(A) + \sum_{m=1}^N \beta_{im} \ln(p_m/M) \\ \alpha_j^*(A) + \sum_{m=1}^N \beta_{jm} \ln(p_m/M) \end{pmatrix}.$$

The remaining $N-3$ v_k necessary to compute the likelihood function value depend on v_i and v_j through the relations given in equations A-8 and A-9. The calculation of this likelihood function requires numerical computation of a general bivariate integral because all of the elements v depend on τ_i and τ_j .

Appendix 2

This appendix sets out a framework for determining the percentage change in long-distance service that should accompany a given percentage change in the price of local service to keep unchanged the total revenue collected from all U.S. households by all local exchange carriers (RBOCs and independents). Using elasticity estimates from the household-level demand modeling effort and from other sources, I determine the range of plausible revenue-neutral two-price-change scenarios analyzed in the paper.

Local exchange carrier revenues are usually classified into four categories: local network services; network access service; long-distance

network services; and other services. Let LR = local network revenues, AR = access revenues, TR = toll revenues, and OR = other revenues, so that TOT , total revenues, satisfies the equation $TOT = LR + AR + TR + OR$. I assume that LR , AR , and TR are functions of the prices of local and long-distance service and that OR does not depend on either price. By definition of rate rebalancing, the percentage change in TOT brought about by a given percentage change in p_l , the price of local service, plus the percentage change in TOT brought about by a given percentage change in p_d , the price of long-distance service, should equal zero.

Taking the derivative of TOT with respect to p_l , yields

$$(B-1) \quad \frac{\partial TOT}{\partial p_l} = \frac{\partial LR}{\partial p_l} + \frac{\partial AR}{\partial p_l} + \frac{\partial TR}{\partial p_l} + \frac{\partial OR}{\partial p_l}.$$

Assuming that OR does not depend on p_l , I can rewrite equation B-1 as

$$(B-2) \quad \frac{TOT}{\partial p_l} \frac{p_l}{TOT} = \frac{\partial LR}{\partial p_l} \frac{p_l}{LR} \frac{LR}{TOT} + \frac{\partial AR}{\partial p_l} \frac{p_l}{AR} \frac{AR}{TOT} + \frac{\partial TR}{\partial p_l} \frac{p_l}{TR} \frac{TR}{TOT}.$$

Let $S_J = (J)R/TOT$, where $J = L, A$, or T , denote the share of total revenue from product J , and

$$\epsilon_{TOT, p_l} = \frac{\partial TOT}{\partial p_l} \frac{p_l}{TOT} \text{ and } \epsilon_{(J)R, p_l} = \frac{\partial (J)R}{\partial p_l} \frac{p_l}{(J)R}$$

denote, respectively, the elasticities of TOT with respect to p_l and the elasticity of revenue from product J with respect to p_l . Using these definitions, equation B-2 can be rewritten as

$$(B-3) \quad \epsilon_{TOT, p_l} = S_{LR} \epsilon_{LR, p_l} + S_{AR} \epsilon_{AR, p_l} + S_{TR} \epsilon_{TR, p_l}.$$

Define

$$\eta_{J, p_l} = \frac{\partial J}{\partial p_l} \frac{p_l}{J}$$

as the elasticity of the quantity demanded of good J with respect to p_l . The following relationships hold between the revenue elasticities and quantity elasticities:

$$(B-4) \quad \epsilon_{LR, p_l} = (1 + \eta_{L, p_l}), \epsilon_{AR, p_l} = \eta_{A, p_l}, \text{ and } \epsilon_{TR, p_l} = \eta_{T, p_l}.$$

The household-level elasticities computed from the model estimates and the CES sampling weights provide one source of estimates of these demand elasticities for all U.S. households. Substituting the relations in equation B-4 into equation B-3 yields:

$$(B-5) \quad \epsilon_{TOT,p_l} = S_{LR}(1 + \eta_{L,p_l}) + S_{AR}\eta_{A,p_l} + S_{TR}\eta_{T,p_l}.$$

Repeating equations B-1 through B-3 for p_d yields:

$$(B-6) \quad \epsilon_{TOT,p_d} = S_{LR}\epsilon_{LR,p_d} + S_{AR}\epsilon_{AR,p_d} + S_{TR}\epsilon_{TR,p_d}.$$

Analogous expressions to those in equation B-4 are:

$$(B-7) \quad \epsilon_{LR,p_d} = \eta_{L,p_d}, \epsilon_{AR,p_d} = \left(\frac{p_d}{p_A} + \eta_{A,p_d} \right), \text{ and } \epsilon_{TR,p_l} = (1 + \eta_{T,p_l}),$$

where p_A is the price of long-distance access. The elasticity of access revenues with respect to p_d embodies the results, discussed in the paper, showing a complete pass-through of a change in the access price into a change in the long-distance price. Assume $p_d = p_A + p_{IXC}$, where p_{IXC} is the price received by the interexchange long-distance carrier for a long-distance call. If all long-distance price reductions come from reductions in the price of access, then

$$(B-8) \quad \frac{\partial p_A}{\partial p_d} = 1 \text{ and } \frac{\partial p_{IXC}}{\partial p_d} = 0.$$

Given that $AR = A \times p_A$, computing the elasticity of AR with respect to p_d using the expressions given in equation B-8 yields the second equation in equation B-7. Substituting equation B-7 into equation B-6 yields

$$(B-9) \quad \epsilon_{TOT,p_d} = S_{LR}\eta_{L,p_d} + S_{AR}\left(\frac{p_d}{p_A} + \eta_{A,p_d}\right) + S_{TR}(1 + \eta_{T,p_d}).$$

Combining equations B-5 and B-9 yields the following expression for the rate rebalancing percentage change in the price of long-distance service that results from a given percentage change in the price of local service:

$$(B-10) \quad \% \Delta p_d = Z(\% \Delta p_l), \text{ where } Z =$$

$$\left[- \frac{S_{LR}(1 + \eta_{L,p_l}) + S_{AR}\eta_{A,p_l} + S_{TR}\eta_{T,p_l}}{S_{LR}\eta_{L,p_d} + S_{AR}\left(\frac{p_d}{p_A} + \eta_{A,p_d}\right) + S_{TR}(1 + \eta_{T,p_d})} \right].$$

According to the most recent annual report from the United States Telephone Association, the 1994 revenue shares for all local exchange carriers are: $S_{LR} = 0.456$, $S_{AR} = 0.318$, and $S_{TR} = 0.134$.³⁶ As discussed in the paper, $p_A = 0.4p_d$, meaning that approximately 40 percent of the price of a long-distance call is paid in access charges, so $p_d/p_A = 2.5$. For the simple case of zero cross-price elasticities, a zero own-price elasticity for local service and -1 as the own-price elasticity of long-distance service along with the above revenue shares yields a value of $Z = -0.96$. This value of Z implies that the revenue-neutral percentage decrease in the price of long-distance service should be a little less in absolute value than the percentage increase in the price of local service.

Using the interior solution model estimates and the CES weights for each households, I can construct estimates of the required aggregate household-level price elasticities. These aggregate elasticities are

$$(B-11) \quad \eta_{L,p_l} = -0.9, \eta_{L,p_d} = 0.05, \eta_{T,p_d} = -2.0, \text{ and } \eta_{T,p_l} = 0.06.$$

Assuming the price elasticity of demand for long-distance is the same, whether it is provided by a local exchange carrier or an interexchange long-distance carrier, implies

$$(B-12) \quad \eta_{A,p_d} = \eta_{T,p_d} \text{ and } \eta_{A,p_l} = \eta_{T,p_l}.$$

Using the elasticities given in (B-11), the ratio of p_d to p_A of 2.5, and the equalities in equation B-12 yields $Z = -1.3$, which implies that the revenue-neutral long-distance price decrease is 1.3 times the percentage increase in the price of local service.

The boundary solution model estimates can be used to construct the following aggregate demand elasticity estimates:

36. United States Telephone Association (1995).

$$(B-13) \quad \eta_{L,p_l} = -0.4, \eta_{L,p_d} = -0.7, \\ \eta_{T,p_d} = -1.75, \text{ and } \eta_{T,p_l} = -0.4.$$

Substituting these elasticities into equation B-10, under the same assumptions as for the interior solution estimates, gives a value for Z of -0.96 , which implies that the revenue-neutral price decrease in long-distance service is a little less in absolute value than the local service percentage increase.

These calculations provide the basis for my chosen combination two-price-change scenarios. All three calculations favor the scenarios calling for price changes in local and long-distance service that are equal but opposite in sign. Because plausible values for the elasticities yield values of Z on the order of -2 , I also consider price changes where the percentage decrease in long-distance service is twice the percentage increase in the local service price. For example, an own-price elasticity of local service of -0.4 and an own-price elasticity of long-distance service of -1.75 combined with zero cross-price elasticities yields $Z = -2$.

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